

**MINISTRY OF  
EDUCATION AND TRAINING**

**VIETNAM ACADEMY OF  
SCIENCE AND TECHNOLOGY**

**INSTITUTE OF PHYSICS AND ELECTRONICS**



**PHAM NGOC DIEP**

**DETECTION AND STUDY  
OF ULTRA HIGH ENERGY COSMIC RAYS**

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**SUPERVISOR: PROF. PIERRE DARRIULAT**

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## PREAMBLE

Cosmic rays are still today, nearly one century after having been discovered, somewhat of a puzzle. In particular the ultra-high energy (UHE) domain [1], which extends up to some  $10^{20}$  eV, is not really understood: Where are the sources? Which is the acceleration mechanism? Which is the nature of these UHE cosmic rays? Is the region beyond the GZK [2] cut-off (resulting from the opening of the pion photoproduction threshold on the cosmic microwave background photons) populated or not?

The current construction and assembly in the Argentina pampas of a giant cosmic ray detector [3], the Pierre Auger Observatory (PAO), is expected to shed significant new light on these questions. For the first time, a hybrid detector will be in operation on a very large scale, considerably reducing the risk of systematic biases toward high energies. The large collection rate will allow for more than one order of magnitude improvement on the size of the UHE data sample: unless extragalactic magnetic fields are unexpectedly high, it should be possible to point back to the brightest of the UHE sources.

The aim of the present work is to clarify some aspects of the methods of cosmic ray detection in the UHE range. The PAO, with which we are associated, is used as an illustration. It measures the transverse shower profile using a ground array of water Cherenkov counters (the surface detector array, SD) and the longitudinal shower profile using a set of eyes (the fluorescence detector, FD). Accurate simulations of these detectors are available from the Auger collaboration but they are not what we are after here. Our aim is to get familiar with some important features of both methods of detection with particular emphasis on those that are less obvious.

The thesis is organized in four chapters: the first chapter deals with general features of cosmic rays and some physics of UHE cosmic rays. Since cosmic ray physics is quite new in Vietnam, I would like to describe it in some detail in this part of the thesis. The second chapter describes the two detection methods used in Auger and the experiment itself. The two last chapters are the main part of the thesis. The third chapter deals with event reconstruction using the surface detector and the fourth chapter with event reconstruction using the fluorescence detector which is considered in some depth. This latter study has been submitted to Vietnam Communication in Physics and has been accepted for publication later this year.

## Chapter 1

### GENERAL INTRODUCTION ON COSMIC RAYS

#### 1.1 A brief history of cosmic ray research

– 1912: On a balloon (Figure 1.1) at an altitude of 5,000 meters, Victor Hess, the founding father of cosmic ray research, discovered "penetrating radiation" coming from space.

– 1932: A debate raged over the nature of cosmic rays. According to a theory of Robert Millikan, they were gamma rays from space hence the name "cosmic rays". But evidence was mounting that cosmic rays were, in fact, mostly energetic charged particles.

– 1933: While watching tracks of cosmic rays passing through his cloud chamber, Carl Anderson discovered antimatter in the form of the anti-electron, later called the positron.

– 1937: Seth Neddermeyer and Carl Anderson discovered the muon in cosmic rays.

The positron and the muon were the first of a series of subatomic particles discovered using cosmic rays – discoveries that gave birth to the science of elementary particle physics. Particle physicists used cosmic rays for their research until the advent of high energy particle accelerators in the 1950's.

– 1938: Pierre Auger, who had installed particle detectors high in the Alps and who was able to use recent development in vacuum tube electronics, noticed that two detectors located many meters apart continued to record coincidences well above the expected random rate. Auger had discovered "extensive air showers (EAS)",



*Figure 1.1 Victor Hess on his balloon in 1912.*

showers of secondary subatomic particles caused by the collision of primary high-energy particles with air molecules. On the basis of his measurements, Auger concluded that he had observed showers with energies of  $10^{15}$  eV – ten million times higher than had been observed before.

– 1949: Enrico Fermi put forth an explanation for the acceleration of cosmic rays. In Fermi's cosmic ray "shock" accelerator, protons speed up by bouncing off moving magnetic clouds in space.

– 1966: In the mid 1960's, Arno Penzias and Robert Wilson discovered that low-energy microwaves permeate the universe. They were immediately identified as the cosmic microwave background (CMB), the remnant radiation from the time when atoms formed, 400,000 yr after the Big Bang. Soon after, Kenneth Greisen, Vadim Kuzmin and Georgi Zatsepin pointed out that high energy cosmic rays would interact with the CMB [2]. The interaction would reduce their energy, so that particles traveling long intergalactic distances could not have energies greater than some  $5 \times 10^{19}$  eV.

– 1991: The Fly's Eye cosmic ray research group in the U.S. observed a cosmic ray event having energy of  $3 \times 10^{20}$  eV. Events with energies in the  $10^{20}$  eV range had been reported during the previous 30 years, but this one was clearly the most energetic.

– 1994: The AGASA group in Japan reported an event with energy of  $2 \times 10^{20}$  eV. The Fly's Eye and AGASA events were higher in energy than any seen before. Where did these two high-energy cosmic rays come from? Neither seemed to point back to an astrophysical object that could produce or accelerate particles of such enormous energies.

– 1995: An international group of researchers begins design studies for a new cosmic ray observatory, the Pierre Auger Observatory (PAO), named in honor of the discoverer of air showers. The new observatory uses a giant array of detectors to detect and measure large numbers of air showers from the very highest energy cosmic rays. Tracing back high energy cosmic rays to their unknown source will advance the understanding of the universe.

## 1.2 Nature and elemental abundances of cosmic rays

For most of the energy range cosmic rays are known to consist of fully ionized atomic nuclei, mostly protons. For example, the typical composition in the region around 100 GeV (where direct measurements are possible from detectors in space as the high rates allow for the use of detectors of a modest size), 99.8% are charged particles, 0.2% are photons or neutrinos. The charged particle composition is: 98% nuclei, 2% electrons and positrons. The nuclear component is made of: 87% p, 12%  $\alpha$ , 0.7%  $Z=6$  to 9, 0.2%  $Z=10$  to 20, 0.1%  $Z=21$  to 30. Cosmic rays' energy density in the universe is of the order of magnitude of the  $\text{eV}/\text{cm}^3$ , similar to those of magnetic fields, of visible light and of CMB photons. However, at variance with those, cosmic rays may carry up to  $10^{20}\text{eV}$  per particle, an enormous energy equivalent to that of a tennis ball moving at 100 km/h.

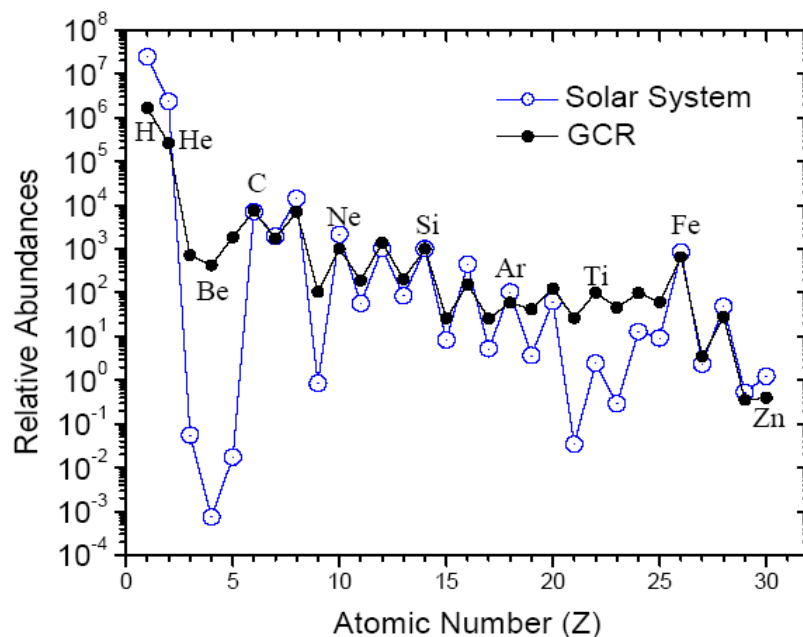


Figure 1.2 Abundance of galactic cosmic rays (full circles). For comparison, the open circles show the abundance of elements in the solar system. The filling of the valleys is understood as being due to spallation reactions.

The galactic cosmic rays (GCR) arriving near the Earth have relative abundances for the more common elements (including C, O, Ne, Mg, Si, Fe, Ni) that are remarkably similar to the relative abundances of these elements in the Solar System (SS). On the other hand, many elements that are rare in the SS (including Li, Be, B, F, Sc, Ti, V) have much higher abundances in the arriving GCR. These

features of the GCR composition, which have been well known for many years, are nicely illustrated<sup>1</sup> in Figure 1.2. The similar abundances of the more common elements is evidence that the composition of the GCR source material, which was accelerated about several  $10^6$  years ago (the time it takes for a typical cosmic ray to wander around in the Milky Way before reaching the Earth), is very similar to that of the cloud that formed the SS some  $5 \times 10^9$  years ago. The fact that the GCR abundances fill in the deep valleys in the SS abundances is understood quantitatively as the result of nuclear interactions of cosmic rays with interstellar gas (spallation reactions). For example, interactions of GCR C, N, O result in fragments of lighter elements,  $^3\text{Li}$ ,  $^4\text{Be}$ ,  $^5\text{B}$ ; similarly interactions of Fe produce fragments of  $^{21}\text{Sc}$ ,  $^{22}\text{Ti}$ ,  $^{23}\text{V}$ .

### **1.3 Energy dependence of the cosmic ray flux**

Cosmic rays have a very broad energy spectrum which extends over 12 decades in energy (from 1 GeV to beyond  $10^{11}$  GeV, hundred million times larger than the highest energy that can be reached with present accelerators) and 32 decades in magnitude of differential fluxes (Figure 1.3).

This spectrum can be well described by a power law:  $dN/dE \sim E^{-\gamma}$ , where the spectral index  $\gamma$  is equal to 2.7 on average. This law is in fact the convolution of the production at the source with the effect of propagation in the interstellar medium. Taking this into account (and taking a typical 10 My journey for a cosmic ray to reach the Earth from its source) one finds a spectral index between 2 and 2.5 rather than 2.7. In fact small variations are observed: indeed, up to around  $10^{15}$  eV the spectrum can be described by a  $\gamma$  of 2.7; by a  $\gamma$  of 3.0 up to around  $10^{18}$  eV; then again by a  $\gamma$  of 2.7 higher up. These two breaks are usually referred to as the “knee” and the “ankle” respectively. Cosmic rays having energy above the ankle region are called ultra-high energy cosmic rays (UHECR). The rate of cosmic rays above this region is extremely small, about 1 *particle/km<sup>2</sup>/year* at the ankle.

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<sup>1</sup> Both GCR and SS abundances are normalized, by convention, to Si =  $10^3$ . GCR abundances for elements  $^3\text{Li}$  -  $^{28}\text{Ni}$  are CRIS (Cosmic Ray Isotope Spectrometer launched aboard NASA's Advanced Composition Explorer in 1997) measurements during solar minimum, August 1997 through April 1998, showing relative fluxes at 170 MeV/nucleon. The GCR abundances for the relatively rare elements  $^{29}\text{Cu}$  and  $^{30}\text{Zn}$  are also from CRIS during solar minimum, but include the full CRIS energy range for these elements, 150 - 550 MeV/nucleon. The GCR abundances for H and He are for solar minimum at 170 MeV/nucleon, derived from measurements by the balloon-borne instrument BESS (Balloon-borne Experiment with a Super-solenoidal Spectrometer) and the GSFC (Goddard Space Fly Center) instrument on the IMP-8 spacecraft.



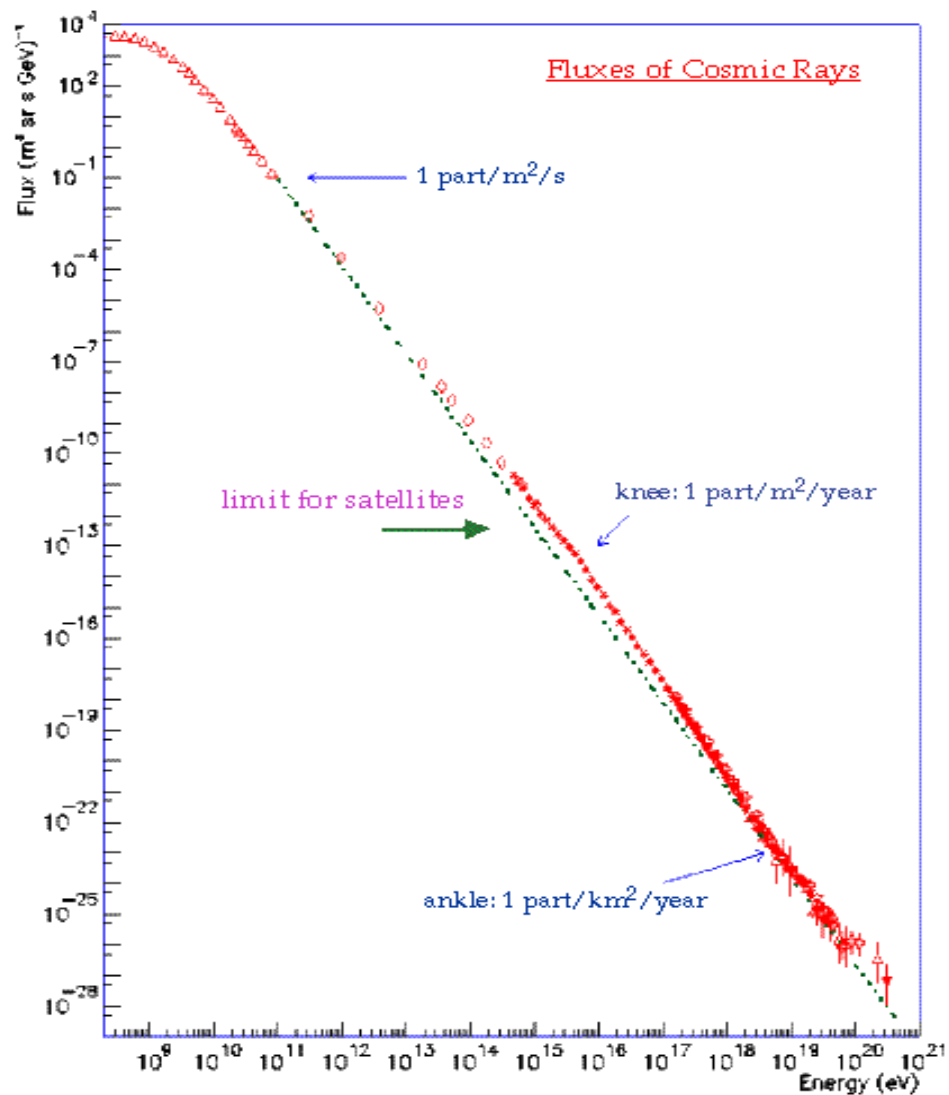


Figure 1.3 All-particle energy spectrum from a compilation of measurements of the differential energy spectrum of CR. The dotted line shows an  $E^{-3}$  power-law distribution for comparison. Approximate integral fluxes are also shown.

Cosmic rays with energy up to the knee (around  $10^{15}$  eV) are believed to be accelerated in the Milky Way by diffusive shock acceleration in supernova remnants (SNR). We still need more data to understand the changes of slope in the spectrum. At the ankle, the cosmic rays' energy spectrum starts receiving contributions (soon dominant) from extra-galactic cosmic rays which have energies high enough to escape from the confinement of the galactic magnetic field. In Figure 1.4 (where the spectrum is multiplied by  $E^3$  for convenience), the breaks at the knee and ankle are clearly visible.

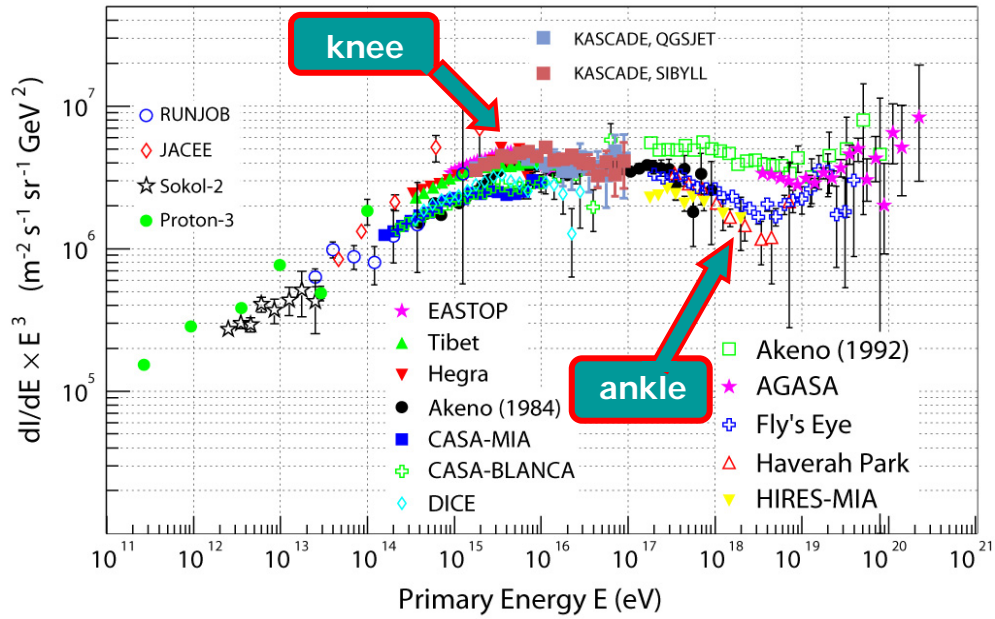


Figure 1.4 A compilation (from data and simulation) of cosmic ray spectrum multiplied by  $E^3$ , showing the knee and ankle.

### 1.4 The GZK effect

The main physics goal of the PAO is a study of the upper end of the energy spectrum. It will provide an accurate measurement of the flux that will settle the GZK question, will hopefully identify the brightest sources and clarify the nature – protons, nuclei, or else – of the UHECRs.

Above  $10^{20}$  eV or so one expects the spectrum to be cut off (this is known as the Greisen-Zatsepin-Kuzmin or GZK cutoff) because of the interactions between the cosmic ray particles with the 2.7K Cosmic Microwave Background (CMB) photons [2]. In the case of incident protons, the resulting distortion of the primary spectrum depends on several parameters: the reaction considered, essentially  $p\gamma \rightarrow \pi^0 p$  or  $\pi^+ n$  either directly or via  $\Delta^+$  production; the distribution of the CMB photons in energy  $\varepsilon$  (Planck spectrum) and angle (isotropic); the cross-section of the reactions being considered; the primary proton spectrum. For a given initial state, defined by the energy  $E$  of the primary proton and the angle  $\theta$  between its momentum and that of the CMB photon, the kinematics of the reaction gives  $M_p \Delta m \cong E\varepsilon(1-\cos\theta)$  where  $\Delta m$  is the difference between the mass of the final state and the proton mass, namely either the pion mass (140 MeV) or the difference between the

mass of the  $\Delta$  resonance and the proton mass (290 MeV). For a given  $\Delta m$ , the distribution in incident energy of the interacting protons is obtained by integration over  $\varepsilon$  and  $\theta$  with weights accounting for the CMB distributions and for the value of the cross-section. Moreover, the final distortion implies another convolution of the proton energy loss over the initial proton spectrum. To get a crude order of magnitude estimate of the effect, we may replace the Planck spectrum by a  $\delta$ -function at  $10^{-3}$  eV, take  $\cos\theta=0$  on average, and assume that any interacting proton is essentially lost (namely that its energy is shifted to a much lower range where its contribution to the spectrum is negligible). This results in an abrupt threshold at  $M_p\Delta m/\varepsilon = 3\times 10^{20}$  eV and  $1.5\times 10^{20}$  eV for  $\Delta$  and  $\pi$  production respectively above which all protons that have interacted are suppressed. The fraction of such protons depends upon the reaction cross-section and the distance of the source to which the target thickness, and therefore the interaction rate, is directly proportional. The exact calculation [4], taking all above factors into account, gives instead distortions illustrated in Figure 1.5. An essential result is that for source distances in excess of 50 Mpc the spectrum is effectively cut-off.

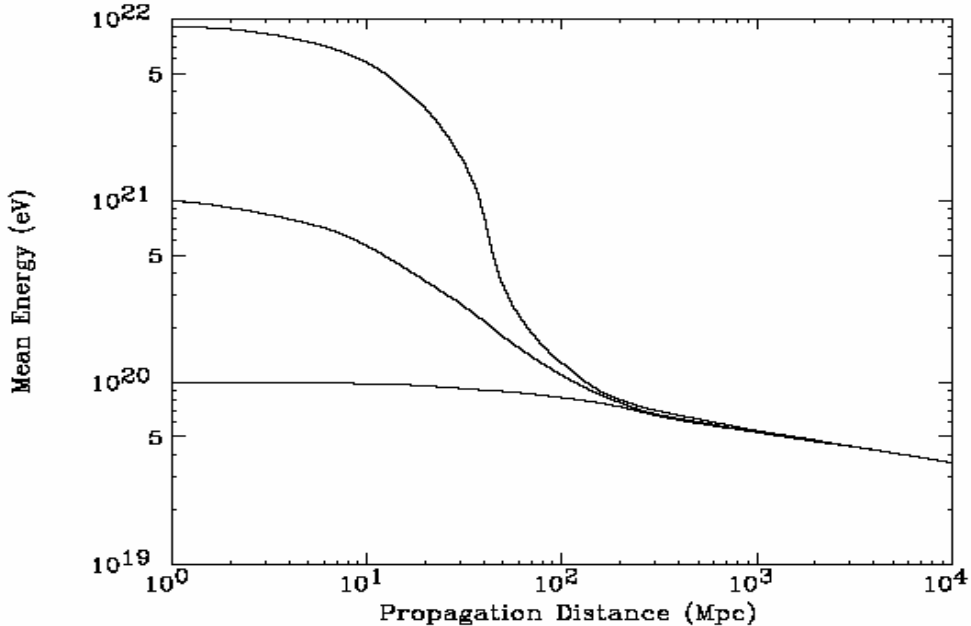


Figure 1.5 Mean energy of protons as a function of propagation distance through the CMB. Curves are for energy at the source of  $10^{22}$  eV,  $10^{21}$  eV,  $10^{20}$  eV respectively.

It is only for sources distant by less than 30 Mpc or so, that the effect of the cut-off is small. The observation of UHE protons above  $10^{20}$  eV or so would

therefore imply that they originate from sources located in the Milky Way or other nearby galaxies in the local cluster, restricting severely the possible acceleration mechanisms.

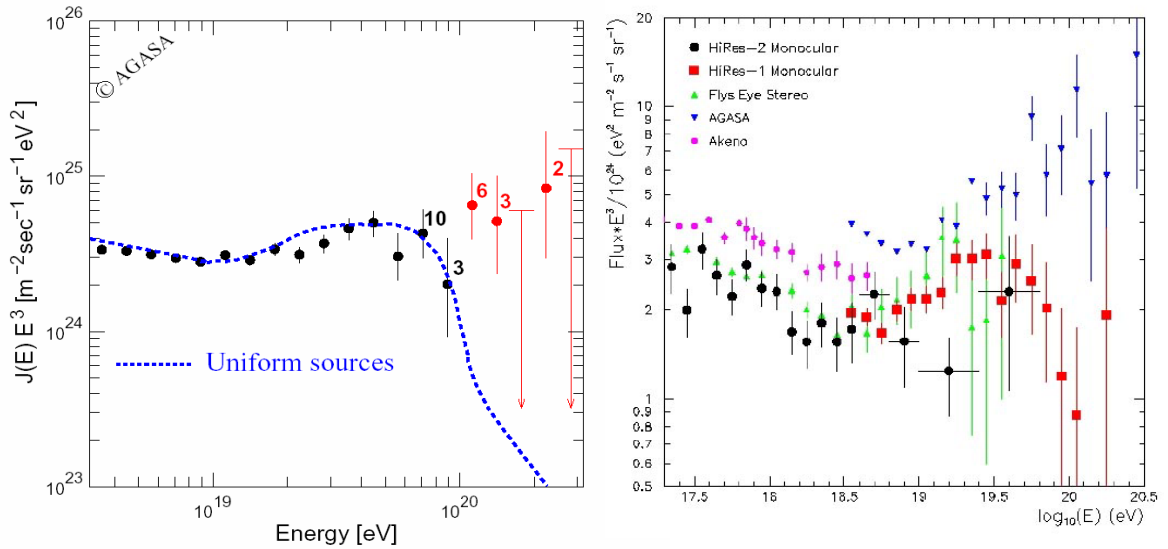


Figure 1.6 Data measured in the region of GZK.

Figures 1.6 and 1.7 show the data observed in the GZK region from different experiments. While the data from HiRes (the full squares in Figure 1.6 on the right) seem to support the GZK prediction, those from AGASA do not. This issue is currently controversial. Recent data from Auger [13] are shown in Figure 1.7 and show that this experiment will soon settle the issue.

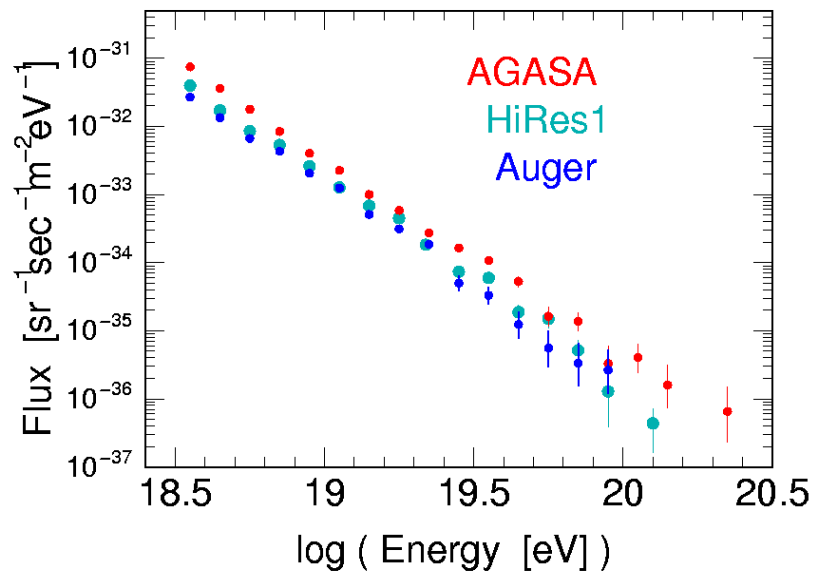
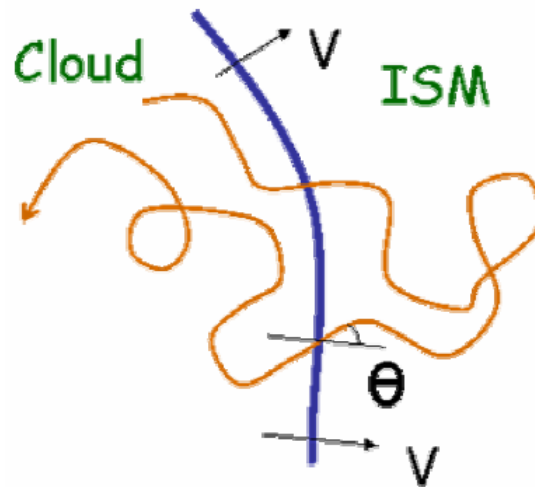


Figure 1.7 Auger data vs Hires1 and AGASA.

### 1.5 Diffusive shock acceleration: an elementary description

Consider a cloud, say a SNR, expanding and moving (at relativistic velocities) into the surrounding inter stellar medium (ISM) (see Figure 1.8). The density of the cloud is supposed to have become so low that the particle mean free paths are large enough for collisions to be always safely ignored. Moreover we assume that both the cloud and the surrounding ISM are fully ionized, say  $H_{II}$  to make it simple, namely electrons and protons. The only relevant interaction between the particles is via magnetic fields. Both the ISM and the cloud have a history from which they have probably inherited some local collective motions sufficient for having trapped some fields. While expanding, the fields have decreased in amplitude to a microgauss scale or so. Original hydrodynamical turbulences have left their imprint in the form of field inhomogeneities. In the process of dilution some field line reconnection has taken place. We may therefore expect a very complex and erratic magnetic field structure in both the cloud and the ISM but we do not really understand how to describe it. We next assume that nearly



*Figure 1.8 Diffusive shock acceleration. The random walk of the accelerated cosmic particle is the result of magnetic fields, not of collisions on ISM particles.*

relativistic particles, say protons, having energies very much higher than the respective energies of the thermal protons of the ISM and cloud (otherwise how could we tell them apart?), are present in the region. We call them “cosmic particles” to distinguish them from the thermal particles. They are the particles that will be accelerated further in the process. We assume that the interface between the cloud and the quiet ISM, called the “shock”, is sufficiently thin to guarantee that cosmic protons entering it at not too large an angle of incidence will traverse it. We

also assume that the magnetic field structures in the ISM and in the cloud are such that a cosmic proton coming from the shock will interact in a random walk with the magnetic field inhomogeneities and have a fair chance to return to the shock and traverse it once more. Two points are essential here. First, in its “interactions” with the magnetic fields, which are nothing more than magnetic bends, the cosmic particle changes momentum but it retains its energy. Second, we can talk of a “cloud rest frame” and of an “ISM rest frame” moving toward each other at velocity  $V$ . A cosmic particle, having energy  $E$  and momentum  $p$  in the rest frame of the medium in which it is, reenters the shock at an incidence  $\theta_{inc}$ . It comes out in the other medium with energy  $E' = \gamma\beta p \cos\theta_{inc} + \gamma E$  with  $\beta = V/c$  and  $\gamma^2 = 1/(1-\beta^2)$ . To first order in  $V/c$ ,  $E' = E + \beta p \cos\theta_{inc}$ , and, as the cosmic particle is relativistic,  $E \sim p$ , hence an energy gain (it is always a gain, never a loss, whatever the direction of the shock traversal)  $\Delta E/E = \beta \cos\theta_{inc}$ . Depending on assumptions  $\langle \cos\theta_{inc} \rangle$  may take different values; a reasonable model gives  $2/3$ . Then, after  $n$  pairs of traversals, the energy has increased to  $E_n = E_0(1 + 4\beta/3)^n$ . However, at some stage, the cosmic particle may ultimately escape the system and acceleration will then stop. Writing  $P_{esc}$  the escape probability (assumed to be E-independent) the fraction of remaining cosmic protons after  $n$  cycles is simply  $(1 - P_{esc})^n$ .

Hence  $dN/dE = -NP_{esc}/(4E\beta/3)$ , namely  $N = N_0 (E/E_0)^{-P_{esc}/(4\beta/3)}$ . This is a power spectrum having an index  $\nu = P_{esc}/(4\beta/3)$ . As  $P_{esc}$  is more or less proportional to  $\beta$ , the spectral index may be expected to take similar values in different configurations. Putting numbers in for the size and age of the SNR one finds that this mechanism is in principle able to accelerate particles up to 100 TeV or so [5]. Its extension to shocks occurring in AGN's might then be able to explain the highest cosmic ray energies, around  $10^{20}$  eV. Multitraversals are essential for the mechanism to work. As they are due to stochastic scattering on field inhomogeneities, it implies that the Larmor radius of the cosmic particles (that of course keeps increasing during the acceleration phase) be large with respect to the shock thickness and at least of the same order of magnitude as the characteristic scale of the magnetic field inhomogeneities (this is necessary for making it meaningful to describe the Brownian motion as occurring in the rest frame of the medium concerned). Note that the ISM and SNR plasmas in which the cosmic particles are moving are themselves subject to proper oscillation modes that may be resonantly excited in particular conditions. Finally one should mention that, like

most other mechanisms that have been studied, diffusive shock acceleration requires that the accelerating site has a large size and is the seat of a large magnetic field. Quantitatively it is the product of these two quantities that defines the maximal attainable energy. This is illustrated in Figure 1.9, usually referred to as the “Hillas plot”.

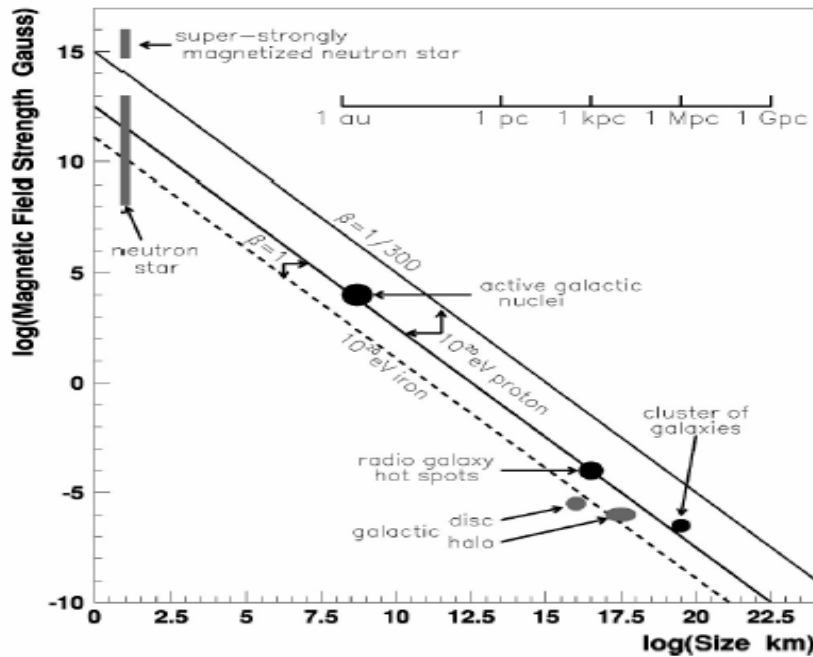


Figure 1.9 Hillas plot showing the minimal value of the product  $BL$  (magnetic field  $\times$  size) for accelerating a proton to  $10^{20}$  eV. The limit is shown as a line in the log-log plot. A few candidate sites are indicated.

## Chapter 2

### DETECTION OF UHECR, THE PIERRE AUGER OBSERVATORY

#### 2.1 Extensive air showers

When a high energy cosmic ray enters the Earth atmosphere it interacts with molecules in the atmosphere and loses energy. The secondary particles produced in the interaction continue to interact with molecules in the atmosphere and create new generations of particles. All the charged particles produced lose energy by ionization (or to a much lesser extent excitation) of the atmospheric molecules. This process repeats until the average energy of secondary particles equals that required to create new particles. At this stage the total number of shower particles reaches its maximum and starts decreasing down to zero. The set of particles produced is called a shower. Such a shower contains three components:

- *Hadronic component*: Each of the interactions of hadronic secondaries (mostly pions and kaons) produces a new generation of high energy hadrons which in turn interact or decay.
- *Electromagnetic component*: At each interaction,  $1/3$  of the incident particle energy goes into neutral *pions*. After a very short time, these *pions* decay into photons that initiate an electromagnetic cascade via pair creation and bremsstrahlung. Note that a very high energy shower, implying a very large number of generations, spends nearly all its energy in the electromagnetic component as  $1/3 + (1/3)(2/3) + (1/3)(2/3)^2 + \dots = 1$ . This however ignores the energy taken away by the third component.
- *Muon and neutrino component*: charged *pions* and *kaons* may decay into *muons* and *neutrinos* if their decay length is short enough in comparison with the interaction length. Muons radiate little energy and have a long lifetime: they often will reach ground. Neutrinos simply escape, undetected.

The shower develops along the trajectory of the primary particle longitudinally and transversally. The shower front is nearly planar, with a small curvature, and moves at the speed of light.





### 2.3 The surface detector

A schematic picture of a surface detector unit is shown in Figure 2.2. It is a Cherenkov counter consisting of a cylindrical molded polyethylene tank, 3.6 m in diameter and 1.55 m high, enclosing a liner filled with 12000 liters of high purity water (1.20 m in height). Contrary to scintillator plates used in other arrays (such as AGASA) these counters offer a cross section to the incident shower particles that has little dependence on the incidence angle.

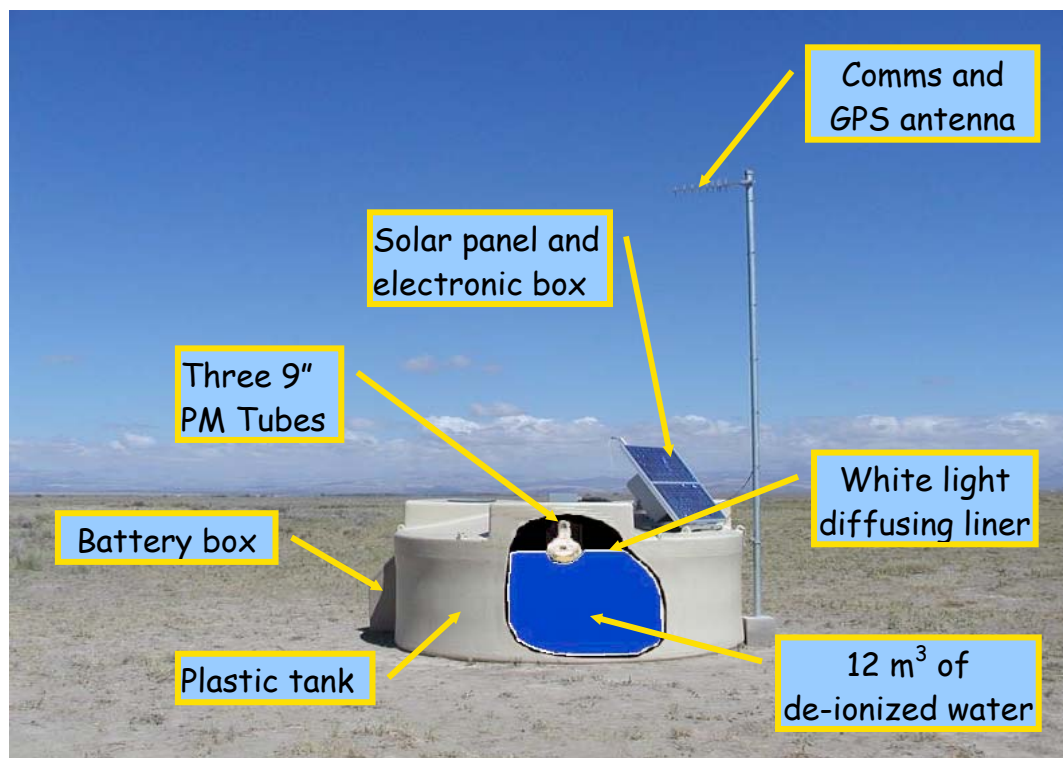


Figure 2.2 Structure of a water Cherenkov counter.

The Cherenkov light is observed through three transparent plastic windows set into the top surface of the liner. Signals from PMTs glued onto the windows are read by local electronics modules. Power is provided by batteries connected to two solar panels, and time synchronization relies on a commercial GPS receiver. A specially designed radio system is used to provide communication between the surface detectors and the central computers of the Observatory. Each tank forms an autonomous unit, recording signals from the ambient cosmic ray flux, independent of the signals registered by any other tank in the surface detector array. A combination of signals clustered in space and time is used to identify a shower. The

SD array has a 100% duty cycle and a well-defined aperture that is independent of energy above  $10^{19}$  eV. Its response is largely independent of weather conditions.

Surface array detectors have been often used to detect cosmic ray showers because of a large number of advantages, including their stability, large detection areas and duty cycle. The arrival time of the particles in the shower can be measured with a good resolution leading to an excellent reconstruction of the primary particle direction ( $\pm 1.5^\circ$ ). Besides, ground array detectors have a well defined aperture resulting in a straight forward determination of their acceptance and of the cosmic ray spectrum.

However, since ground array experiments detect a sample of the shower development at a single depth, the energy reconstruction can not be derived in a direct or calorimetric way. In fact, the energy of the primary particle must be reconstructed from the lateral distribution of particles in the shower. In 1969, Hillas et al. [6] showed that at a given distance of the shower axis, the fluctuations of the lateral distribution function (LDF) due to intrinsic shower fluctuations are minimized. In subsequent works [7], it was also shown that the measured signal fluctuation is very small at a certain distance in spite of the different types of primary particles and of models used to define the LDF.

The distance at which the fluctuations reach their minimum is a convolution of the intrinsic shower-to-shower fluctuation, the experimental uncertainties and the reconstruction procedures. One very important experimental input is the array spacing which mathematically determines the properties of the LDF fit [8]. The Haverah Park experiment [9] used water Cerenkov tanks and determined the distance of least fluctuation to be 600 meters. Operating with the same ground array technique, the Pierre Auger Observatory [3] has determined the distance of least fluctuation to be 1000 meters. Therefore, the LDF in Auger has the form [3]:

$$S(r(\text{km})) = S(1000)r^{-\nu}, \text{ for zenith angle } < 60^\circ, \text{ where } \nu = 4.08 - 1.26 \sec \theta.$$

$S(1000)$ , the signal at 1000 m from the shower axis, is used to estimate the primary energy. The relationship between the energy in EeV ( $1 \text{ EeV} \equiv 10^{18} \text{ eV}$ ) and  $S(1000)$  currently adopted in Auger is

$$E(\text{EeV}) = 0.12(\sqrt{1 + 11.8(\sec \theta - 1)^2} S(1000))^{1.05}$$

## 2.4 The fluorescence detector

### 2.4.1 The PAO eyes

The first fluorescence detectors were two fly's eyes [10] that were operated in Utah between 1981 and 1992. Together, they consisted of some hundred mirrors looking at the whole sky and focusing light on arrays of photomultiplier tubes, over a thousand in total.

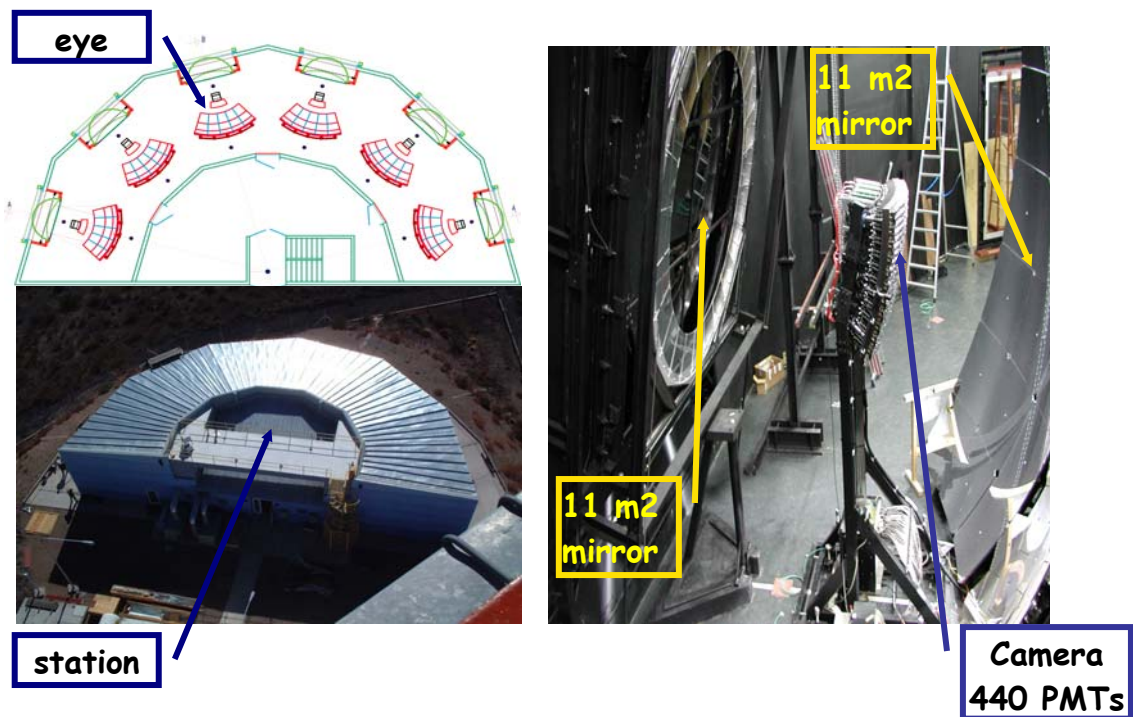


Figure 2.3 Auger Fluorescence Detector

The experience gained in this pioneering work made it possible to design optimally its successors: HiRes [11] and the PAO [3]. While HiRes consists again of two fly's eyes, separated by a distance of 12.5 km, the Auger eyes (shown in Figure 2.3) match the configuration of the surface detector array in order to optimize the efficiency of hybrid detection: four stations of six eyes each are located on the periphery of the array. Each eye covers a field of view of approximately  $28^\circ \times 30^\circ$ , from  $62^\circ$  to  $90^\circ$  in zenith angle and  $180^\circ$  in azimuth for each station. An eye includes a UV filter and shutter to protect the detector, a  $3.5 \times 3.5 \text{ m}^2$  spherical mirror and an array of 440 photomultiplier tubes at its focus. The phototubes have hexagonal photocathodes closely packed together and special light guides allowing

for negligible light loss at their junctions. Each phototube (pixel) has a field of view of approximately  $1.4^\circ \times 1.4^\circ$  and its signal is recorded in a fast analogue to digital converter in 100 ns slices. This latter feature is essential in achieving a high accuracy. Additional equipment is used to measure and monitor the light attenuation in the atmosphere (the attenuation length is of the order of 15 km) and to calibrate and monitor the light collection efficiency of the detector. Figure 2.4 shows the pixel patterns observed by two successive eyes as well as the shower geometry.

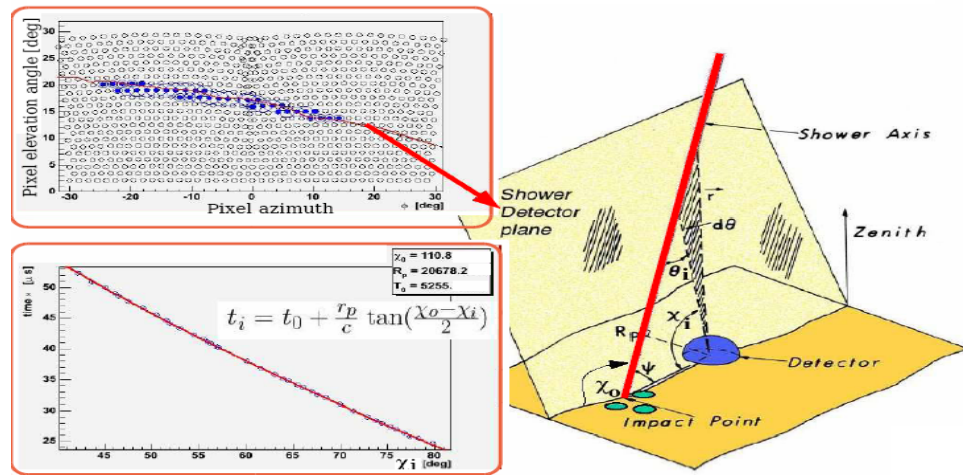


Figure 2.4: On the left, the upper panel shows an example of pixel patterns observed in two successive eyes. In the lower panel, the measured pixel time is plotted as a function of observed angles. On the right, the picture shows the shower geometry.

## 2.4.2 Longitudinal development of an air shower

The longitudinal development of an air shower is described by its profile. On average, the longitudinal profile of a shower induced by a given primary cosmic ray is a universal function of the depth of atmosphere traversed. It is well described by a Gaisser Hillas function [11].

$$S(X) = S_{\max} \left( \frac{X - X_1}{X_{\max} - X_1} \right)^{(X_{\max} - X_1)/w} e^{-(X_{\max} - X)/w} \quad (1.1)$$

where  $S(X)$ ,  $X_1$ ,  $S_{\max}$ ,  $X_{\max}$  and  $w$  are the shower size at depth  $X$ , the depth of the first interaction, the maximum value of the shower longitudinal profile, the depth at which this maximum is reached and a quantity close to the interaction length for the

primary particle, usually taken equal to  $70 \text{ g/cm}^2$ , respectively. On average, the difference  $X_{max} - X_I$  depends on the energy and on the nature of the primary,  $X_I$  depends only on the nature of the primary.  $X_{max}$  increases logarithmically with energy in GeV [12]:

$$X_{max} = 107 [\text{g/cm}^2] + 67 [\text{g/cm}^2] \times \log_{10} E \quad (1.2)$$

This correlation is illustrated in Figure 2.5 obtained from experiments and simulations. For the development of showers initiated by a nucleus of energy  $E$  and mass  $A$ , it is found that  $X_{max}$  is approximately expressed [14] as  $X_{max} = (1-B)X_0(\ln E/\varepsilon - \langle \ln A \rangle)$ , where  $X_0$  is the radiation length in air ( $37.1 \text{ g/cm}^2$ ),  $\varepsilon$  is the critical energy in air (81 MeV),  $B$  is 0 for a pure electromagnetic cascade and less than 1.0 by an amount that depends on the hadronic interaction model for a hadronic cascade; in practice, at a given energy,  $X_{max}(p) - X_{max}(Fe) \sim 100 \text{ g/cm}^2$  (as shown in Figure 2.6).

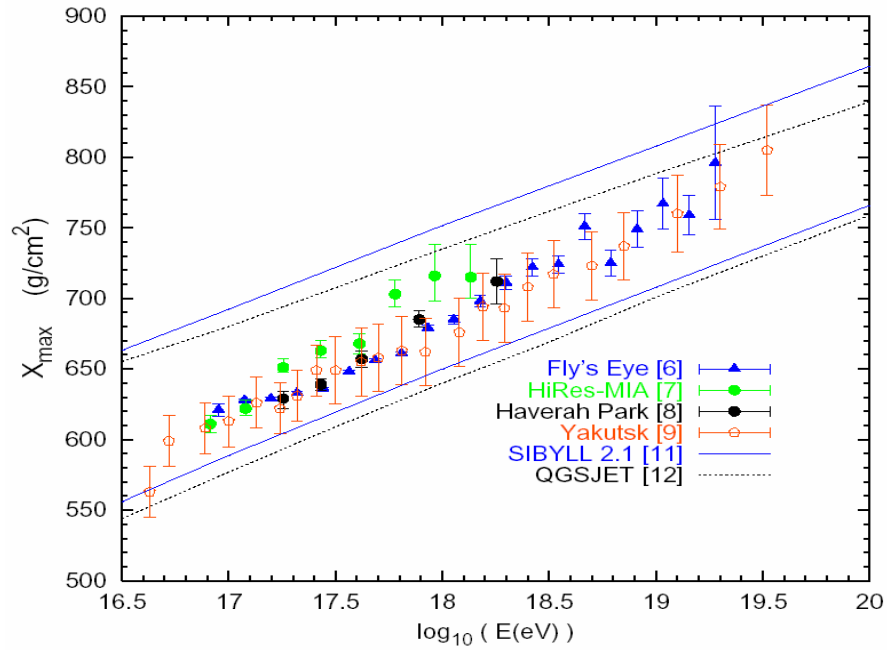


Figure 2.5 The correlation between  $X_{max}$  and  $\log_{10} E$ ,  $E$  being the energy of the primary cosmic ray, as obtained from various experiments and simulations.

The longitudinal profiles of proton-showers (red) and iron-showers (blue) obtained from a sample of simulated events at  $10^{19} \text{ eV}$  are shown in Figure 2.6. Larger fluctuations are obtained with primary protons and a shift in  $X_{max}$  of about  $100 \text{ g/cm}^2$  is clearly visible.

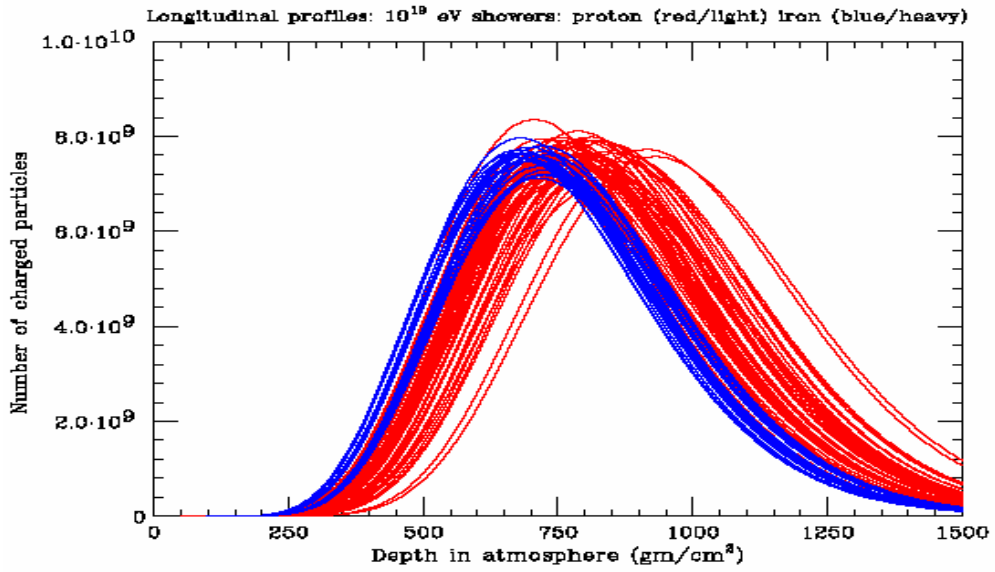


Figure 2.6: Longitudinal profiles of simulated proton (red) and iron (blue) showers with a primary energy of  $10^{19}$  eV.

The energy carried by the electromagnetic component ( $E_{em}$ ) is evaluated by the track length integral as

$$E_{em} = \frac{(\text{critical energy of electron in air})}{(\text{radiation length of electron in air})} \times (\text{total track length in g/cm}^2)$$

$$= (2.18 \text{ MeV}) \times (\text{total track length in g/cm}^2)$$

The fraction of energy passed to the electromagnetic component for hadronic showers varies with primary energy and mass and is about 80–90% of the total energy for  $10^{19}$  eV. In the case of proton primaries, the number of electrons at maximum shower development fluctuates by about 10% from shower to shower. The total track length is determined by the integral  $\int N_e(x) dx$ , making an assumption about the shape of the cascade curve after the observation level. Allowing for lost energy (e.g. neutrinos) and particles traveling not quite parallel to the shower axis, the primary energy has been estimated [1] as

$$E = (2.65 \text{ MeV}) \times \int N_e(x) dx.$$

To estimate the energy resolution of the shower as measured by the fluorescence method, the following factors must be taken into account [10]: the air fluorescence efficiency, the subtraction of direct and scattered Cherenkov light, the attenuation and scattering of photons, and uncertainties in the geometrical reconstruction.

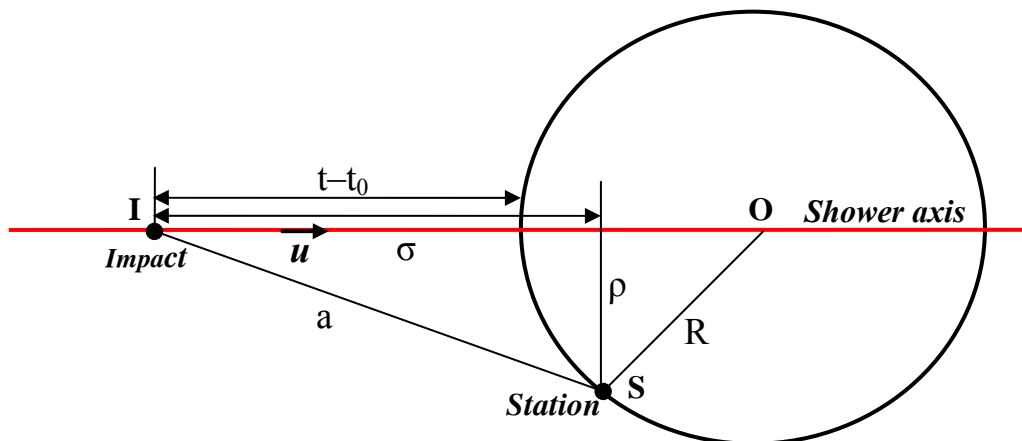
## Chapter 3

### EVENT RECONSTRUCTION USING THE SURFACE DETECTOR

We have written a program to reconstruct the direction and the front curvature of Auger showers. To test this reconstruction program, a Monte Carlo has been written that: generates showers and calculates the response of the Auger array. By this way, the reconstructed shower parameters and the generated shower parameters can be compared and the quality of the reconstruction program can be evaluated.

#### 3.1 Geometric reconstruction

The shower geometry is described in simple terms by seven parameters: the space-time coordinates of the impact on ground of the shower front on the shower axis  $(x_0, y_0, z_0, t_0)$ , the zenith angle  $\theta$  and azimuth  $\varphi$  of the unit vector  $\vec{u}$  directed along its axis, and the curvature  $C$  of the shower front assumed to be spherical and infinitely thin.



*Figure 3.1 Definition of parameters used to describe an air shower. The drawing is in the plane containing the station and the shower axis. O and S are the center of the sphere and the station respectively and I is the impact on ground.*

In a first step the space coordinates of the impact point,  $(x_0, y_0, z_0)$ , are reconstructed as the coordinates of the centre of gravity of the stations hit using the



signals as weights. In a second step, a finer tuning can be made by searching for a best fit (using the radial and energy dependence of the lateral distribution function) at the same time as evaluating the energy. Here, however, we concentrate on the first step and the problem reduces to the evaluation of  $\theta$ ,  $\varphi$ , (or equivalently  $\lambda = \sin\theta \cos\varphi$  and  $\mu = \sin\theta \sin\varphi$ ),  $t_0$  and  $C$  from the times of arrival  $t_i$  of the signal in at least four of the stations. From Figure 3.1, calling  $\sigma_i$  the projection on the shower axis of the vector joining the impact to station  $i$

$$\vec{u}=(\lambda, \mu, \cos\theta) \quad (3.1)$$

$$\sigma_i=a_i.\vec{u}=(x_i-x_0)\lambda+(y_i-y_0)\mu \quad (3.2)$$

where  $x_i$  and  $y_i$  are the coordinates of the detection stations (located on a flat horizontal ground at  $z_0=0$ , at the nodes of a triangular lattice with mesh size equal to 1.5 km).

Calling  $\rho_i$  the distance of a station hit to the shower axis we have

$$\rho_i^2=a_i^2-\sigma_i^2=(x_i-x_0)^2+(y_i-y_0)^2-\sigma_i^2 \quad (3.3)$$

The basic relationship between  $t_i$  and the parameters of the showers is obtained from Figure 3.1 by splitting the segment OI in two different ways :

$$t_i-t_0+R=\sigma_i+\sqrt{R^2-\rho_i^2} \quad (3.4)$$

where  $R=1/(2C)$  is the radius of the shower front.

Squaring (3.4) we obtain

$$([t_i-t_0]-\sigma_i)^2+2R([t_i-t_0]-\sigma_i)+\rho_i^2=0 \quad (3.5)$$

or equivalently

$$t_i=t_0+\sigma_i-C[(\sigma_i-[t_i-t_0])^2+\rho_i^2] \quad (3.6)$$

$$\text{Writing } \alpha_i=[(\sigma-t)^2+\rho^2]_i \quad (3.7)$$

we find

$$t_i=t_0+\lambda(x_i-x_0)+\mu(y_i-y_0)+C(-\alpha_i) \quad (3.8)$$

Equations (3.8), one for each station hit, allow for the evaluation of the four unknown shower parameters  $t_0$ ,  $\theta$ ,  $\varphi$  and  $C$  as long as there are at least 4 stations hit.

For  $C = 0$ ,  $t_i$  is a linear function of  $\lambda$ ,  $\mu$  and  $t_0$ . In that case, they can be calculated analytically by minimizing  $\chi^2 = \sum_i [t_i - t_0 - \lambda(x_i - x_0) - \mu(y_i - y_0)]^2 / (\Delta t_i)^2$ .

Here, for simplicity, we take the measurement error  $\Delta t_i$  to be a constant (arbitrary taken as unity). However for  $C \neq 0$ , the  $\alpha_i$ 's being functions of  $\lambda$ ,  $\mu$  and  $t_0$ , the equations are no longer linear. But as  $C = 0$  is a good first order approximation, it is still possible to solve equation (3.8) by iteration: set  $C_0 = 0$ , solve (3.8) analytically in  $\lambda$ ,  $\mu$  and  $t_0$ , calculate  $\alpha_{i0}$  using Relation (3.7), solve (3.8) analytically in  $\lambda$ ,  $\mu$ ,  $t_0$  and  $C$  with  $\alpha_i = \alpha_{i0}$ , calculate  $\alpha_i = \alpha_{i1}$  by putting the new values of  $\lambda$ ,  $\mu$ ,  $t_0$  and  $C$  into the Relation (3.7), iterate until it converges. In fact, the convergence is very rapid.

### 3.2 Monte Carlo simulation

In order to illustrate and check the procedure of geometric event reconstruction described above, a rudimentary simulation program (toy Monte Carlo) has been written.

In a first step shower axes are generated isotropic in space. This means with uniform  $\varphi$  and  $\cos\theta$  distributions (between  $-\pi$  and  $\pi$  and between  $0.36 = \cos 69^\circ$  and  $1.0$  respectively). The curvature  $C = 1/(2R)$  of the shower front is taken with a uniform distribution between  $0$  and  $0.1 \text{ km}^{-1}$ . The detector array covers a square area of  $30 \times 30 \text{ km}^2$ . Impacts are generated uniformly in a smaller concentric square,  $20 \times 20 \text{ km}^2$ , leaving a  $5 \text{ km}$  safety margin all around. The event time  $t_0$  is taken with a uniform distribution between  $0$  and  $10 \text{ km}$  (all distances and times are measured in  $\text{km}$ ).

In a second step, the response of the detector array is calculated according to the arithmetic described in the preceding paragraph. The arrival time of each station hit is deduced from the generated values of the parameters  $\sigma$ ,  $\rho$ ,  $C_0$ . The signal of each station hit is calculated by assuming that the particle density on the shower front decreases exponentially with  $\rho$ , namely  $S_i = \exp(-\rho_i^2/\rho_0^2)$  with  $\rho_0^2 = 2 \text{ km}^2$ . The number  $N_s$  of stations hit is calculated by requiring that their signals be greater than a given threshold.

In a third step, the impact is reconstructed as the centre of gravity of the stations hit using the signals as weights. In this same step, the shower parameters

are calculated in planar and curved approximation from Relations (3.7) and (3.8), by iteration as described above.

The fitted values of the four shower parameters  $\theta$ ,  $\varphi$ ,  $t_0$  and  $C$  were found identical to the generated values, providing evidence for the good performance of the iteration method described above. Figures 3.2a, 3.2b, 3.3a and 3.3b compare generated and reconstructed values of the impact coordinates for each of two threshold values, 10% and 1% the signal at the shower core. As expected a much better accuracy is achieved for the lower value of the threshold. Also shown in the figures are the number  $N_s$  of stations hit (above threshold) and its dependence on  $\theta$ . Here again the results agree with expectation: there are on average 12.4 stations hit at high threshold and 24.3 stations hit at low threshold. Moreover, the number of stations hit increases trivially with  $1/\cos\theta$ .

This program has been used to reconstruct events collected using the four Cherenkov tanks installed on top of the laboratory roof as well as to study the systematic errors attached the energy measurement using the Auger surface detector array. However, these studies are beyond the scope of the present thesis and will not be described here.

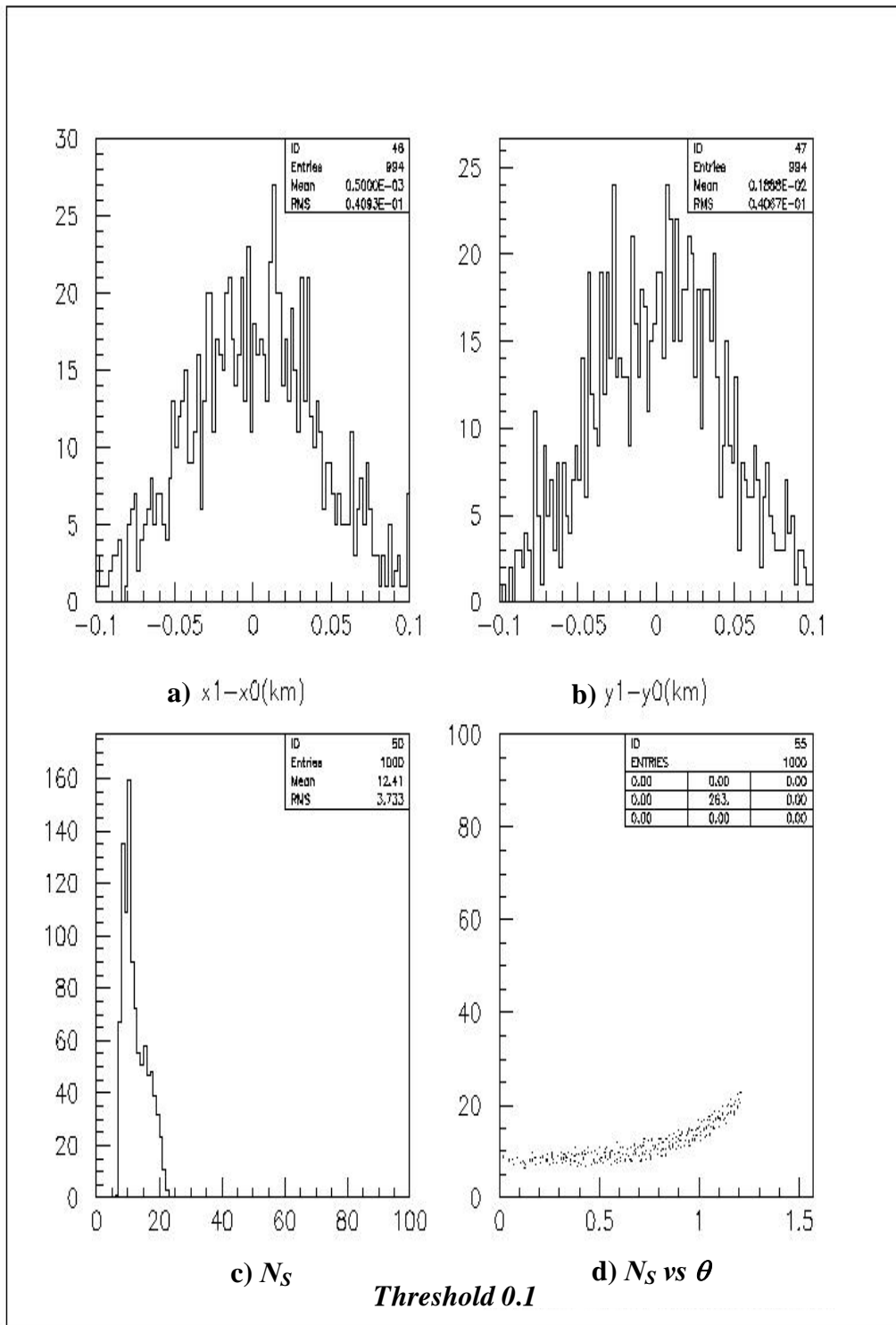


Figure 3.2: Threshold=0.1 a and b: distributions of the differences between reconstructed and generated impact coordinates. c and d: distribution of the number of stations hit and its dependence on  $\theta$ .

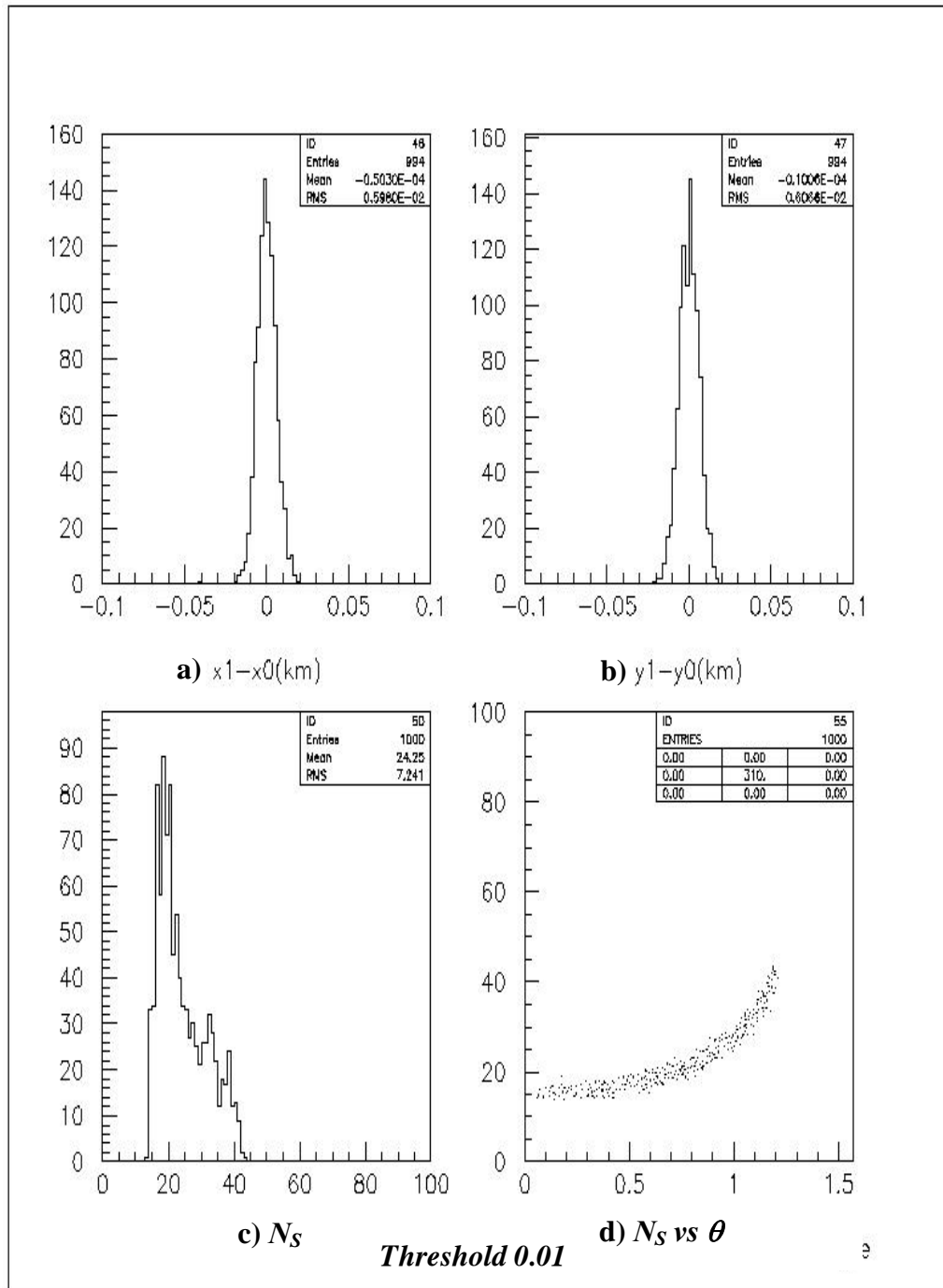


Figure 3.3: Threshold=0.01 a and b: distributions of the differences between reconstructed and generated impact coordinates. c and d: distribution of the number of stations hit and its dependence on  $\theta$ .

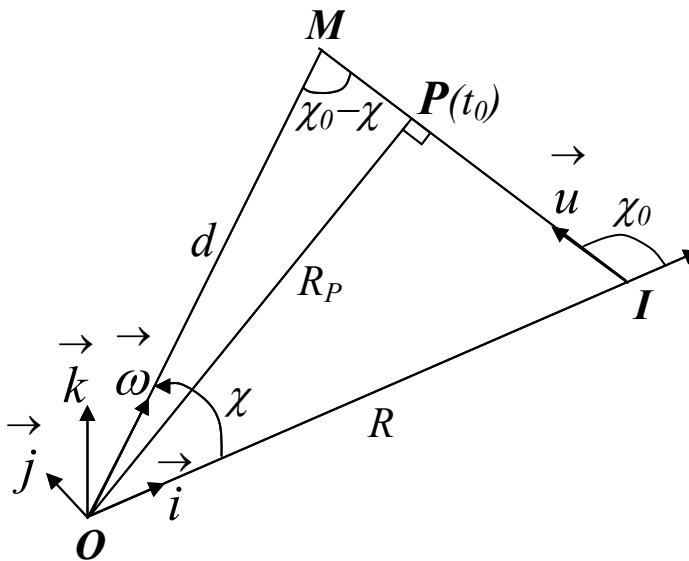
## Chapter 4

### EVENT RECONSTRUCTION USING THE FLUORESCENCE DETECTOR

A program has been written and tested in order to study and understand the main features of the fluorescence detector and its response to showers. In order to do this we have developed a code that simulates the detector.

#### 4.1 Shower reconstruction: principle of the method

A single eye is able to measure both the location of the shower axis in space and the shower energy. Locating the position of the shower axis in space is made in two steps. The first, straightforward step consists in fitting a line to the pattern of pixels hit: this fixes directly the position of the shower detector plane (SDP, defined in Figure 4.1) with an accuracy that scales with the pixel aperture.



*Figure 4.1: The eye  $O$  looks at the shower axis  $IM$  that crosses ground in  $I$ .  $OIM$  is the SDP. The unit vectors along  $OI$ ,  $IM$  and  $OM$  are called  $\mathbf{i}$ ,  $\mathbf{u}$  and  $\boldsymbol{\omega}$  respectively. The unit vector along the vertical is called  $\mathbf{k}$ .  $OP=R_p$  is the shortest distance from the eye to the shower axis.  $OM=d$  and  $OI=R$ . The shower front passes  $P$  at time  $t_0$  and  $M$  at time  $t$ . The angles  $\chi$  and  $\chi_0$  are defined as  $(\mathbf{i}, \boldsymbol{\omega})$  and  $(\mathbf{i}, \mathbf{u})$  respectively.*

The second step, locating the shower axis in the SDP, is far more delicate and relies on the time measurement. Each 100 ns slice centered on time  $t_i$  detects an amount of light  $\Delta S_i$  emitted from a small shower segment spanning an atmosphere thickness  $\Delta a_i$  centered on an average thickness  $a_i$  as measured from the direction of view of the associated pixel(s). The position of the shower axis in the SDP is defined by two parameters:  $R_p$  and  $\chi_0$ ;  $R_p$  is the shortest distance from the eye to the shower axis and

$\chi_0$  is the angle between the shower axis and the trace OI of the SDP on ground. Similarly, the axis of view of the pixel(s) associated with mean time  $t_i$  is defined by the angle  $\chi_i$  that it makes with this trace.

The time taken by the light to reach the eye is

$t_i = t_0 - MP + MO = t_0 - R_p / \tan(\chi_0 - \chi_i) + R_p / \sin(\chi_0 - \chi_i)$ , leading to the relation:

$$(t_i - t_0) / R_p = 1 / \sin(\chi_0 - \chi_i) - 1 / \tan(\chi_0 - \chi_i) = \tan([\chi_0 - \chi_i] / 2) \quad (4.1)$$

Here,  $t_0$  fixes the absolute time of arrival of the shower front in P and is of no interest to the present problem (it is however essential for binocular or hybrid detection in order to combine signals from different detectors): locating the shower axis in the SDP depends only on the time differences between different pixel signals. We see immediately from Relation (4.1) that the parameters  $t_0$  and  $R_p$  are essentially measured respectively by a shift and a scaling in ordinate of the  $t_i$  vs  $\chi_i$  relation, leaving the determination of  $\chi_0$  to rely on the measurement of its shape, in particular its curvature. As, in practice, this curvature is very small, the measurement of  $\chi_0$  is particularly delicate. We also see from Relation (4.1) that, to the extent that the range spanned by  $\chi_i$  is small,  $R_p$  and  $\chi_0$  are anticorrelated, the product  $R_p \tan([\chi_0 - \langle \chi_i \rangle] / 2)$  being equal to  $\langle t_i \rangle - t_0$ .

Once  $R_p$  and  $\chi_0$  are known,  $a_i$  and  $\Delta a_i$  are easily calculated and the associated amount of fluorescence light is obtained by scaling the measured signal  $\Delta S_i$  by the reciprocal of the detection solid angle and correcting it for the attenuation in the atmosphere, both operations depending on the distance  $d_i = OM$  between the eye and the shower segment under consideration.

We see from Figure 4.1 that  $d_i = R_p / \sin(\chi_0 - \chi_i)$  and that the altitude  $z_i$  of M above ground is 
$$z_i = d_i \vec{\omega} \cdot \vec{k} = d_i \sin \chi_i \sin \Psi = R_p \sin \chi_i \sin \Psi / \sin(\chi_0 - \chi_i) \quad (4.2)$$

where  $\Psi$  is the angle between the SDP and ground (measured from the trace of the shower in the pixel plane). The relation between pressure  $p_i$  and altitude above sea level,  $z_i + z_{ground}$ , is known from atmospheric standard data. Here, for simplicity, we assume that it is a simple exponential of the form

$$p_i = p_{sl} \exp(-[z_i + z_{ground}] / z_{atm}) \quad (4.3)$$

with a sea level pressure  $p_{sl}$  of 1000 g/cm<sup>2</sup> and  $z_{atm} = 7.8$  km.

Combining relations (4.2) and (4.3) we obtain

$$a_i = p_i / \cos(\vec{u}, \vec{k}) = p_i / (\sin\Psi \sin\chi_0). \quad (4.4)$$

Having reconstructed the longitudinal shower profile, and extrapolated it outside the field of view when necessary, its integral is in principle proportional to the energy, the proportionality factor being obtained from shower simulations including state of the art knowledge of the laws obeyed by its development. In practice, it is convenient to fit the observed profile to some standard profile that depends essentially on two parameters, the atmosphere thickness  $a_{first}$  at which the first interaction occurs and that,  $\Delta_{max} + a_{first}$ , at which the shower reaches its maximum development. While  $\Delta_{max}$  depends directly on energy,  $a_{first}$  is an indicator of the nature of the primary: heavier nuclei initiate their shower earlier than lighter nuclei do. Such a particularly simple standard profile is given by [11] as

$$\ln\{\Delta S_i / \Delta a_{ij}\} = \ln\{E / \Delta_{max}\} + (\Delta_{max} / w) \ln\{[a_i - a_{first}] / \Delta_{max}\} + (\Delta_{max} - [a_i - a_{first}]) / w \quad (4.5)$$

Here,  $E$  is the energy in GeV,  $w=70$  g/cm<sup>2</sup> and  $\Delta_{max}$ , the thickness separating the first interaction from the shower maximum, is parameterized as a linear function of the logarithm of the energy [12] as

$$\Delta_{max} = 107 \text{ g/cm}^2 + 67 \text{ g/cm}^2 \times \log_{10} E. \quad (4.6)$$

In principle, the shower energy can be obtained from  $\Delta_{max}$  as:  $E = 10^{**}\{(\Delta_{max} - 107)/67\}$ . However, this determination is far less precise than that obtained from the integral of the shower profile. Note that in the PAO  $z_{ground} = 1.4$  km, implying that a  $10^{20}$  eV vertical shower reaches maximum development on ground.

Finally,  $a_{first}$  has an exponential distribution with an interaction length  $\Lambda_{int}$  calculated from a proton-air interaction cross-section  $\sigma_{int}$  parameterized [12] as a function of energy as

$$\sigma_{int} = 150 \text{ mb} + 40 \text{ mb} \times \log_{10} E. \quad (4.7)$$



## 4.2 Monocular observation: locating the shower axis in the SD plane

### 4.2.1 The method

In the present section we study in some detail the properties of Relation (4.1) that is used to calculate the position of the shower axis in the SDP in case of monocular observation. For simplicity we consider showers located in a vertical SDP ( $\Psi=90^\circ$ ) and assume that they cross the whole field of view (from  $\chi_{up}=28^\circ$  to  $\chi_{down}=0^\circ$ ). Their energy is irrelevant to the present considerations. The only variable parameters are therefore  $R_P$  and  $\chi_0$ . In order to compare showers that are at similar distances from the eye, we fix their distance in the middle of the field of view ( $\chi_{mean}=14^\circ$ ) to be  $R \sin \chi_0 / \sin(\chi_0 - \chi_{mean}) = 10$  km. Finally we assume that times are exactly measured in bins of 100ns. For each shower we obtain this way a set of  $(t_i, \chi_i)$  pairs which we may modify at will in order to simulate measurement errors. To calculate  $R_P$ ,  $t_0$  and  $\chi_0$  from these data we minimize their  $\chi^2$  to Relation (4.1). Writing  $u_i = \tan(\chi_i/2)$  and  $u_0 = \tan(\chi_0/2)$ , Relation (4.1) reads  $(t_i - t_0)(1 + u_i u_0) = R_P (u_0 - u_i)$  that can be rewritten as

$$\{t_0 + R_P u_0\} + u_i \{t_0 u_0 - R_P\} - t_i u_i \{u_0\} = t_i \quad (4.8)$$

Relation (4.8) is linear in the parameters inserted in curly brackets: its  $\chi^2$  can be minimized exactly by solving the linear system of three equations expressing that its derivative with respect to each of the three parameters cancels.

### 4.2.2 Effect on the $\chi_0$ and $R_P$ measurements of systematic errors on $\chi_i$

Shifting all  $\chi_i$ 's by a same quantity  $\delta$  obviously results in an identical shift for  $\chi_0$ , the other parameters being unaffected. Similarly, shifting each  $\chi_i$  by a quantity  $\delta_i = \delta (1 - \chi_i / \chi_{mean})$  results in small changes in both  $R_P$  and  $\chi_0$ : the ratio  $\Delta R_P / R_P \delta$  cancels for  $\chi_0 = 104^\circ$  (showers normal to the mean axis of view) and reaches  $-4$  at  $\chi_0 = 30^\circ$  and  $3$  at  $\chi_0 = 150^\circ$ ; the ratio  $\Delta \chi_0 / \delta$  is negative and of the order of  $-3$  over the whole field of view.

However, shifting each  $\chi_i$  by a quantity  $\delta_i = \delta (1 - \chi_i / \chi_{mean})^2$  results in dramatic changes in both  $R_P$  and  $\chi_0$ , approximately proportional to  $\delta$ . This is illustrated in Figure 4.2 where the relative shifts  $\Delta R_P / R_P \delta$  and  $\Delta \chi_0 / \delta$  are shown as a function of  $\chi_0$ . The shifts in  $\chi_i$  are amplified by very large factors reaching over 30 in the  $R_P$  case and over 60 in the  $\chi_0$  case.

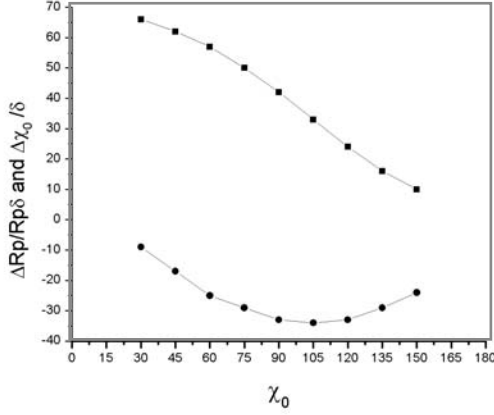


Figure 4.2: Dependence of  $\Delta R_p / R_p \delta$  (circles) and  $\Delta \chi_0 / \delta$  (squares) on  $\chi_0$  (degrees) for quadratic errors of the form  $\delta_i = \delta(1 - \chi_i / \chi_{mean})^2$ .

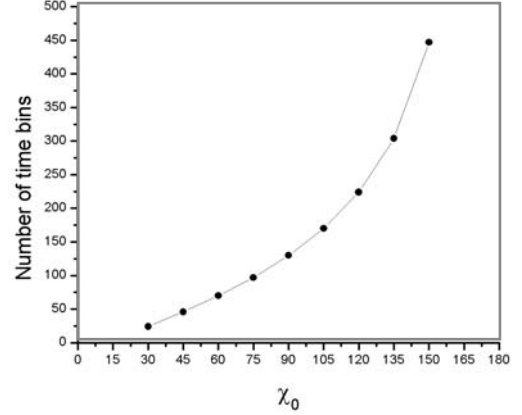


Figure 4.3: Dependence on  $\chi_0$  of the number  $N_{bin}$  of 100ns time bins spanned by the shower.

This extreme sensitivity to errors having a quadratic dependence on  $\chi$  is the dominant feature of Relation (4.1). It is easily understood by developing it around  $\chi_{mean}$ :

$$(t-t_0)/R_p = \frac{\{ \tan((\chi_0 - \chi_{mean})/2) - \tan((\chi - \chi_{mean})/2) \}}{\{ 1 + \tan((\chi_0 - \chi_{mean})/2) \tan((\chi - \chi_{mean})/2) \}}$$

Defining  $\tau_0 = \tan((\chi_0 - \chi_{mean})/2)$  and  $\tau = \tan((\chi - \chi_{mean})/2)$  we have to second order in  $\tau$ :

$$(t-t_0)/R_p = (\tau_0 - \tau)(1 - \tau_0 \tau + [\tau_0 \tau]^2) = \tau_0 - (1 + \tau_0^2) \tau + \tau_0 (1 + \tau_0^2) \tau^2$$

$$\text{Namely, in the plane } (t, \tau), \quad t = [t_0 + R_p \tau_0] - [R_p (1 + \tau_0^2)] \tau + [R_p \tau_0 (1 + \tau_0^2)] \tau^2 \quad (4.9)$$

In the linear approximation, we only measure  $t_0 + R_p \tau_0$  and  $R_p (1 + \tau_0^2)$ : we need to measure the curvature to disentangle  $R_p$  from  $\tau_0$ .

Defining a slope  $S = -R_p (1 + \tau_0^2)$  and a curvature  $C = R_p \tau_0 (1 + \tau_0^2)$ , we have  $\tau_0 = -C/S$  and  $R_p = -S^3 / (S^2 + C^2)$ . The constant term does nothing but to define  $t_0$ . For zero curvature, the slope measures  $R_p$ . And for a small curvature ( $C \ll S$ ), the slope measures essentially  $R_p$  and the curvature measures  $\tau_0$ . In particular, any source of error that may introduce a spurious curvature in Relation (4.9) will directly affect the measurement of  $\tau_0$  and therefore of the other two variables.

Note that for  $R_p = 0$ , namely when the shower points to the eye, all the light reaches the eye at a same time  $t_0$  and hits a same pixel: this is the signature of  $R_p = 0$  and  $\chi_0$  is simply equal to the  $\chi$  of that pixel in this case. Indeed the time span,  $t(\chi_{down}) -$

$t(\chi_{up})$ , depends strongly on  $\chi_0$  as illustrated in Figure 4.3. The smaller it is, the poorer the  $\chi_0$  measurement.

As a further illustration of the properties of Relation (4.1) we have studied the effect of uncorrelated Gaussian errors having an *rms* value  $\delta$  (but truncated at  $\pm 0.7^\circ$  in order to stay within a pixel size). In spite of the errors being random, large shifts result in both  $R_P$  and  $\chi_0$ . The rms value of the  $\Delta\chi_0/\delta$  distribution is found to be approximately inversely proportional to the number  $N_{bin}$  of 100ns bins spanned by the shower with  $N_{bin} \times Rms(\Delta\chi_0/\delta) \sim 1300$ . Figures 4.4 and 4.5 illustrate the case of a vertical shower ( $\chi_0=90^\circ$ ): Figure 4.4 shows the correlation between  $\Delta R_P/R_P \delta$  and  $\Delta\chi_0/\delta$  and Figure 4.5 shows the distribution of  $\Delta\chi_0/\delta$ . Not only very large errors may occur – in the case of  $\chi_0$  typically 50 times as large as the average error on  $\chi_i$  – but there is also a systematic shift on both the  $R_P$  and  $\chi_0$  measurements.

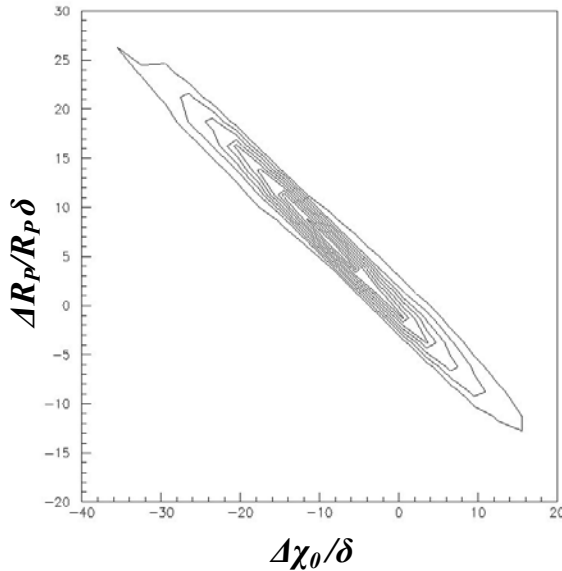


Figure 4.4: Correlation between  $\Delta R_P/R_P \delta$  (ordinate) and  $\Delta\chi_0/\delta$  (abscissa) for  $\chi_0=90^\circ$  and for Gaussian truncated errors (see text).

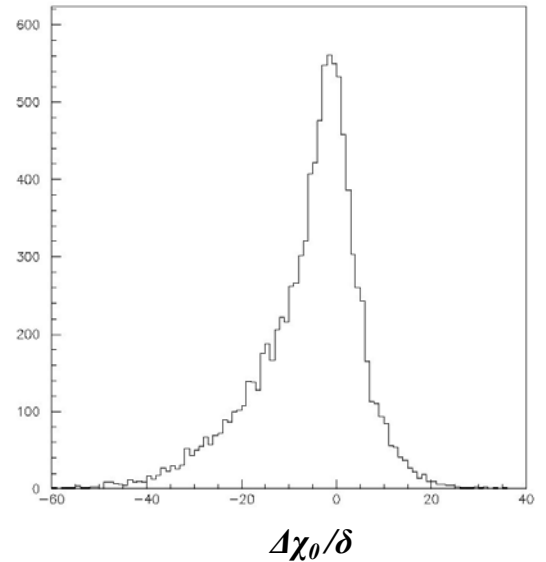


Figure 4.5: Distribution of the shifts  $\Delta\chi_0/\delta$  induced by Gaussian errors for  $\chi_0=90^\circ$  (see text).

### 4.2.3 Resulting distortions of the shower profile

By differentiating Relation (4.2) with respect to  $R_P$  and  $\chi_0$  we can study the distortion of the shower profile resulting from errors on the location of the shower

axis within the SDP. For simplicity we assume that the SDP itself is exactly reconstructed. We obtain:

$$dz_i/z_i = dd_i/d_i = dR_P/R_P - \cotan(\chi_0 - \chi_i) d\chi_0 \quad (4.10)$$

and from Relations (4.3) and (4.4)  $da_i/a_i = dp_i/p_i - \cotan \chi_0 d\chi_0 = -dz_i/z_{atm} - \cotan \chi_0 d\chi_0$

$$da_i/a_i = -[z_i/z_{atm}] dR_P/R_P + ([z_i/z_{atm}] \cotan(\chi_0 - \chi_i) - \cotan \chi_0) d\chi_0 \quad (4.11)$$

There are two kinds of distortions induced by errors on  $R_P$  and  $\chi_0$ : distortions of the scale of atmospheric thicknesses (abscissa) and distortions of the light signal (ordinate). The former are directly obtained from Relation (4.10) while distortions of the light signal result from errors on the detection solid angle and on the atmospheric light attenuation, themselves induced by errors on the measurement of  $d$ .

Distortions of the atmospheric thickness scale are constrained by their ground value,  $[da/a]_{ground} = -\cotan \chi_0 d\chi_0$ . On ground an error on  $R_P$  causes a parallel translation of the shower axis and induces therefore no error on  $a$  while an error on  $\chi_0$  trivially does as  $a$  is inversely proportional to  $\sin \chi_0$ . As a result, such distortions will only be important when large  $z$  values are in the field of view, namely for very inclined showers pointing in the eye direction. However, they will have little effect on the energy measurement to the extent that the shower profile is well contained within the field of view: in such cases the energy measurement is dominated by the integrated light signal, *i.e.* by the first term in Relation (4.5), that is independent of the atmospheric thickness scale. Only in the case where the longitudinal profile needs to be extrapolated outside the field of view will such errors be of relevance to the energy measurement. On the contrary, the measurement of  $a_{first}$  is directly affected by distortions of the atmospheric thickness scale in the region of early shower development, which are again important in the case of inclined showers pointing toward the eye.

Distortions of the light signal are instead of direct relevance to the energy measurement. The measured signal is inversely proportional to  $d^2$  and has an additional exponential dependence on  $d$  due to light attenuation in the atmosphere. A 1% error on  $d$  induces therefore an energy error in excess of 2%. On ground, Relation (4.10) reads  $dd/d = dR_P/R_P - \cotan(\chi_0) d\chi_0$ .

For a vertical shower, the relative error on  $d$  equals that on  $R_p$  while for inclined showers the two terms of Relation (4.10) may add or subtract depending whether the shower points away from the eye or toward it. When the errors on  $R_p$  and  $\chi_0$  have a same origin and are anticorrelated, as in the case of the showers discussed in Section 4.2.2, the error on  $d$  is more important for inclined showers pointing to the eye.

### 4.3 Measuring the shower energy

An accurate measurement of the shower energy relies on an accurate measurement of the longitudinal shower profile and on a precise knowledge of the relation between energy and amount of fluorescence light. Which is more important depends on what one is after. For example, in the GZK region, establishing whether or not there is a cut-off does not require an excellent knowledge of the energy scale but does require an excellent control over systematic errors. If there is such a cut-off and if one has enough statistics and an excellent control over systematic errors, one will see it and one might then argue with some mischief that it could be used to calibrate the energy scale in a region where accelerator data are lacking. Without going that far, we simply wish to underline the need for an excellent control over systematic errors.

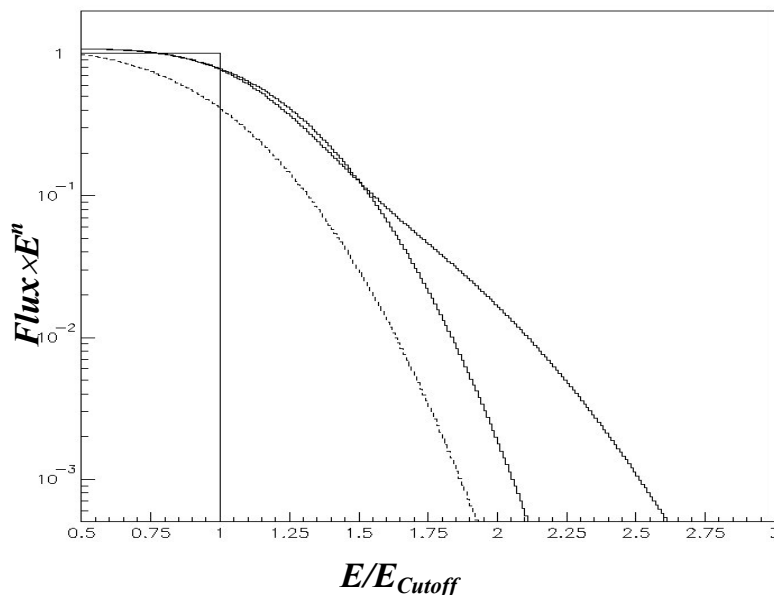


Figure 4.6 : Simulation of measured fluxes (times  $E^n$ ) for an initial  $E^{-n}$  spectrum abruptly cut off at  $E_{cutoff}=1$  (see text) as a function of energy relative to cutoff. The full lines are for  $n=3$  and 30% errors, either single Gaussian (narrower curve) or double Gaussian (wider curve). The dotted line is for  $n=0$  and single Gaussian errors.

We illustrate this point by considering an  $E^{-n}$  spectrum abruptly cut-off at  $E_{cutoff}$  and showing how it can be distorted by systematic errors. Figure 4.6 shows simulated experimental spectra multiplied by  $E^n$  obtained for measurement errors having an *rms* value of 30%. For  $n=3$ , the error distribution is represented by a single Gaussian in one case, by the sum of two Gaussians, one twice as large as the other, in the other case. To have less than a 1% probability to measure an event of a given energy or higher, one needs to go to  $f=1.58$  times the cut-off in the single Gaussian case and to  $f=1.76$  times the cut-off in the double Gaussian case, illustrating the effect of tails in the error distribution. The dotted line is for  $n=0$ , a step function. Here,  $f=1.37$ : at variance with the  $n=3$  case, the smearing is smaller, illustrating a well known property of steeply falling spectra. These trivial remarks underline the importance of understanding well the shape of the error distribution and the danger of simply assuming that it is a Gaussian.

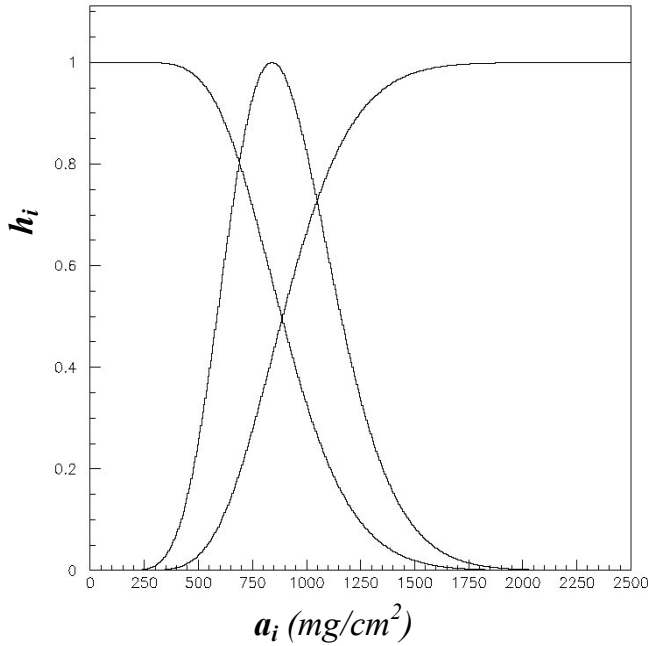


Figure 4.7: The shower profile used in the present study and the fractional losses  $f_{early}$  and  $f_{late}$  and as functions of the atmosphere thickness traversed ( $mg/cm^2$ ).

$=\Delta_{max}$ ). Its dependence on  $a$  is illustrated in Figure 4.7 together with the fractional losses  $f_{early}$  and  $f_{late}$ . For a given field of view, a sliding window of width  $w_{field}$  and starting at thickness  $a_{field}$ , we calculate the  $\chi^2$  describing the deviation between the

A potential source of systematic errors is the need for an extrapolation of the observed profile outside the field of view. This may be studied using a profile of the form (4.5) and assuming that only a portion of the profile is observed, leaving unobserved energy fractions  $f_{early}$  and  $f_{late}$  corresponding respectively to the early and late parts of the shower development. The profile used here,  $h_i$  vs  $a_i$ , is of a shower of  $10^{20}$ eV ( $\Delta_{max} = 840g/cm^2$ ). The  $h_i$  scale is normalized to have  $h_i=1$  at maximum (where  $a$

undistorted profile ( $\Delta_{max}$ ,  $a_{first}$ ) and a distorted profile ( $\Delta_{max} + \delta\Delta_{max}$ ,  $a_{first} + \delta a_{first}$ ). The error ellipse  $\chi^2=1$  gives the errors  $\Delta(a_{first})$  and  $\Delta(\Delta_{max})$  on the measurements of  $a_{first}$  and  $\Delta_{max}$  as well as the correlation between them. These errors are a measure of the quality of the measurement and of its sensitivity to the width and location of the sliding window. They are shown in Figures 4.8 and 4.9 as a function of  $a_{field}$  for three values of  $w_{field}$ ,  $100\text{g/cm}^2$ ,  $200\text{g/cm}^2$  and  $300\text{g/cm}^2$ . Their qualitative behavior does not much depend on the exact value of the uncertainties used in the calculation of the  $\chi^2$ . Here they were taken of the form  $\Delta h_i = 0.001 + 0.01h_i$  (remember that  $h$  is normalized to unity at maximum). The features visible on Figures 4.8 and 4.9 are easily understood: the errors on  $a_{first}$  and  $\Delta_{max}$  decrease when the width  $w_{field}$  of the field of view increases; the error on  $a_{first}$  becomes larger when moving away from the early part of the shower development; the error on  $\Delta_{max}$  is minimal when the field of view covers the shower maximum.

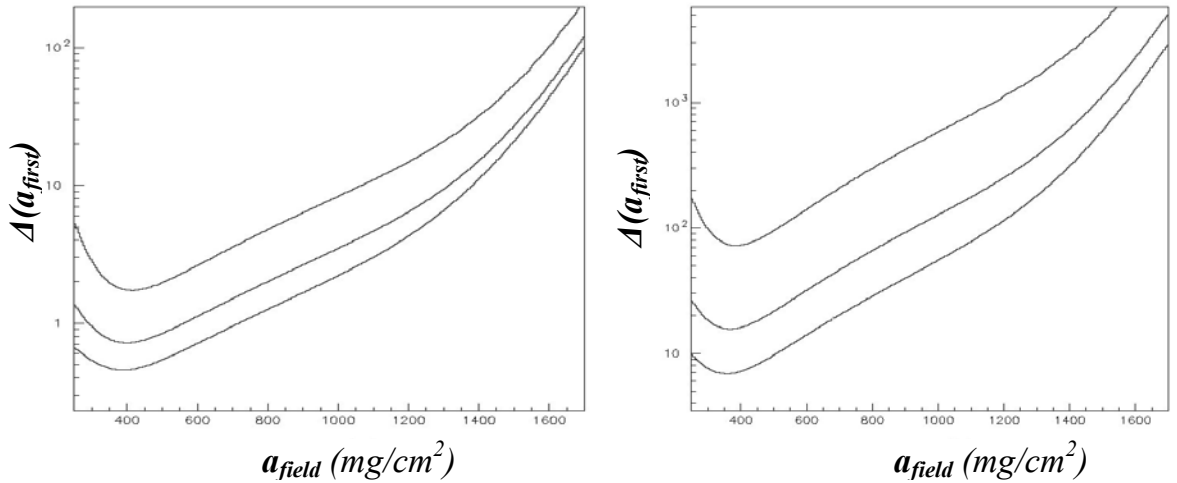


Figure 4.8 Dependence of  $\Delta(a_{first})$  on  $a_{field}$  for  $w_{field} = 100$  (up),  $200$  (middle) and  $300$  (down)  $\text{g/cm}^2$ . On the left, both the shape and the intensity of the profile are considered, on the right only the shape.

The error on  $\Delta_{max}$  is usually small because  $\Delta_{max}$  is essentially fixed by the intensity of the light and depends only weakly on the shape of the profile. On the contrary, the error on  $a_{first}$  is typically an order of magnitude larger because the measurement of  $a_{first}$  is not directly affected by the  $h$  scale but only by the shift of the profile along the abscissa.

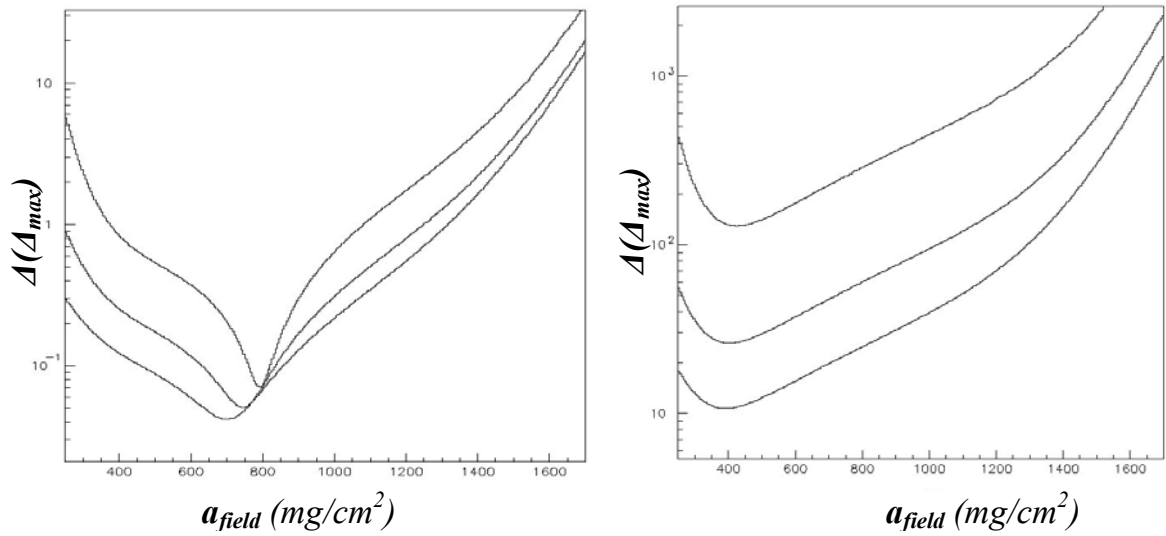


Figure 4.9 Dependence of  $\Delta(\Delta_{max})$  on  $a_{field}$  for  $w_{field} = 100$  (up), 200 (middle) and 300 (down)  $g/cm^2$ . On the left, both the shape and the intensity of the profile are considered, on the right only the shape.

It is interesting to evaluate the deterioration of the accuracy when one considers only the shape of the profile and disregards its intensity. The results of such calculations are shown on the right parts of Figures 4.8 and 4.9. The error on  $\Delta_{max}$  increases by two orders of magnitude, illustrating the comment that was just made, and that on  $a_{first}$  by one order of magnitude.

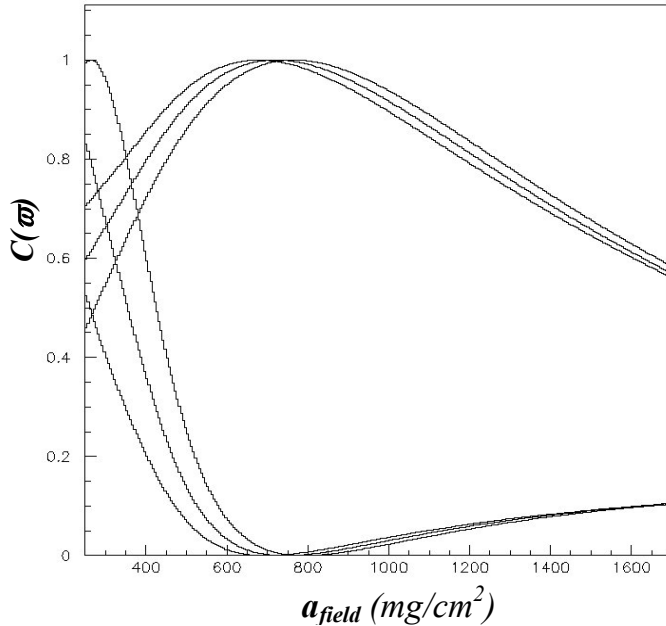


Figure 4.10 Correlation  $C(\varpi)$  between the measurements of  $a_{first}$  and  $\Delta_{max}$ . For the three lower curves, both the shape and the intensity of the profile are considered, for the three upper curves only the shape.

$\Delta_{max}$  increases by two orders of magnitude, illustrating the comment that was just made, and that on  $a_{first}$  by one order of magnitude.

Finally, Figure 4.10 illustrates the correlation between the two measurements. It shows the dependence of  $C(\varpi) = \sin^2(2\varpi)$  on  $a_{field}$  where  $\varpi$  defines the orientation of the major axis of the error ellipse.  $C$  is a measure of the correlation between the two measurements. When both the



intensity and the shape of the profile are considered the correlation is weak except for low values of  $a_{field}$  when the field of view covers only the early part of the shower development. In particular, when the field of view covers the maximum of the shower development,  $\Delta_{max}$  is essentially fixed by the intensity measurement and  $a_{first}$  is then obtained from the measurement of the value of  $a$  at which the maximum occurs: in such a case there is no correlation between the two measurements. On the contrary, when the field of view covers the early part of the shower development, an increase of  $\Delta_{max}$  is easily compensated by an increase of  $a_{first}$ . When the shape of the profile is considered alone the correlation is much stronger and reaches a maximum when the field of view covers the maximum of the shower development. Indeed, for a locally flat profile, one can choose  $\Delta_{max}$  as one wishes and  $a_{first}$  is then exactly determined: the two measurements are maximally correlated.

#### **4.4 Hybrid detection**

The elementary considerations presented in the preceding sections have shown that the field of view should offer a broad coverage in both time span and atmosphere thickness traversed in order to achieve an accurate measurement. Broad time coverage is essential in monocular observation; however, even so, monocular measurements remain of much lesser quality than binocular or hybrid measurements.

Binocular detection allows for an accurate positioning of the shower axis when the two SDP's make a large angle: the time information becomes almost irrelevant in such a case. However, many other sources of uncertainties that are inherent to the fluorescence detection method, such as errors in the light-energy calibration and the attenuation in the atmosphere, are still at play. For this reason we restrict the present discussion to the case of hybrid detection: in addition of allowing for an accurate location of the shower axis, it offers a completely independent measurement of the shower energy with very different sources of possible systematic errors. This feature makes hybrid detection particularly attractive.

The question we ask in the present section is to which extent are fluorescence and surface detections compatible, can hybrid events be accurately measured in both detectors? Indeed, it might be that showers benefiting of a broad time and thickness coverage in the fluorescence detector would fall outside the acceptance of the surface detector array (SDA). As in the previous sections, we do not attempt a

precise and detailed description of the problem but aim simply at identifying the relevant parameters and understanding the main features. For simplicity, following the broad lines of the PAO design, the SDA is taken to cover a circle having a radius of 30 km and four equidistant eyes are located on its circumference and look toward its centre. The field of view of each eye covers  $180^\circ$  in azimuth,  $28^\circ$  in elevation and the depth of field is 30 km. A shower is considered to be detected by the SDA if and only if its axis impacts ground within the area of the circle. In practice, any shower detected in the SDA is seen in one of the four eyes. It is therefore sufficient to study showers that are seen in one of the eyes. Their energy is fixed at  $10^{20}$ eV and their zenith angle  $\theta$  is taken to have a uniform cosine distribution.

Figure 4.11 shows distributions of the distance  $\rho$  of the shower impact on ground to the centre of the SDA circle measured in units of its radius. Namely showers having  $\rho < 1$  are detected by the SDA and showers having  $\rho > 1$  are not.

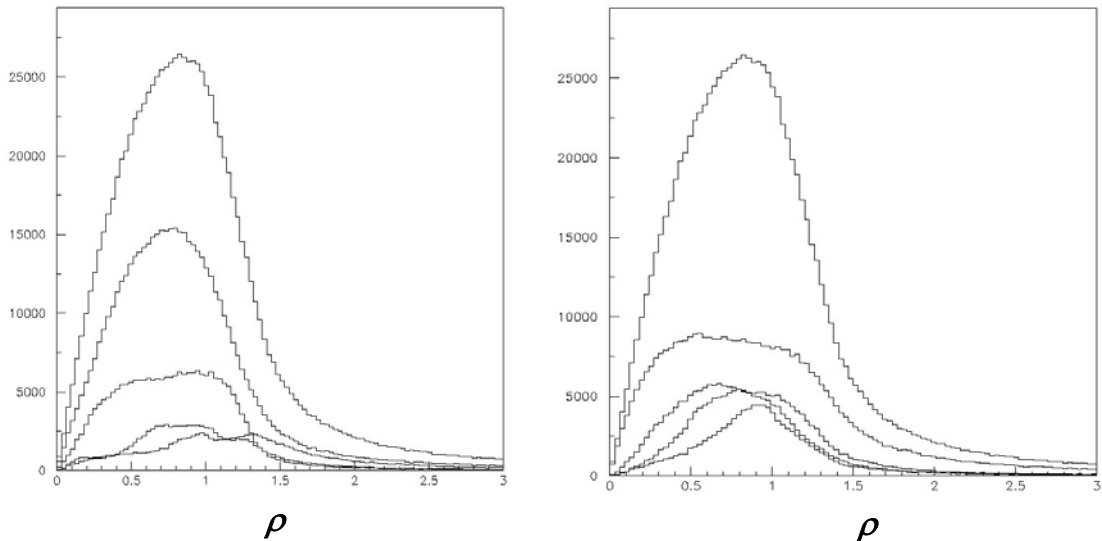


Figure 4.11a  $\rho$  distributions of various shower samples. From top to bottom: all showers,  $w_{field} > 800 \text{g/cm}^2$ , between 600 and  $800 \text{g/cm}^2$ , between 400 and  $600 \text{g/cm}^2$  and  $< 400 \text{g/cm}^2$ .

Figure 4.11b  $\rho$  distributions of various shower samples. From top to bottom: all showers,  $t_{field} > 13 \text{km}$ , between 7 and 13 km, between 3 and 7 km and  $< 3 \text{km}$ .

About 2/3 of the events seen in the eye are also seen in the SDA and are therefore hybrids. Also shown in Figure 4.11 are  $\rho$  distributions of various sub samples of the generated showers. Figure 4.11a is for different slices of  $w_{field}$ , the atmosphere

thickness covered by the field of view, and Figure 4.11b is for different slices of  $t_{field}$ , the time span covered by the field of view.

Figure 4.12 shows the  $\cos(\theta)$  and  $\varphi$  distributions, where  $\theta$  and  $\varphi$  are the zenith angle and respectively azimuth of the shower axis, for showers having  $\rho < 1$  and  $\rho > 1$  respectively. Most of the showers that are detected by the eye but miss the SDA have large zenith angles and preferably fly toward the eye, that is have small values of  $\cos(\theta)$  and  $\varphi$  values that are enhanced around  $0^\circ$  and depressed around  $180^\circ$ .

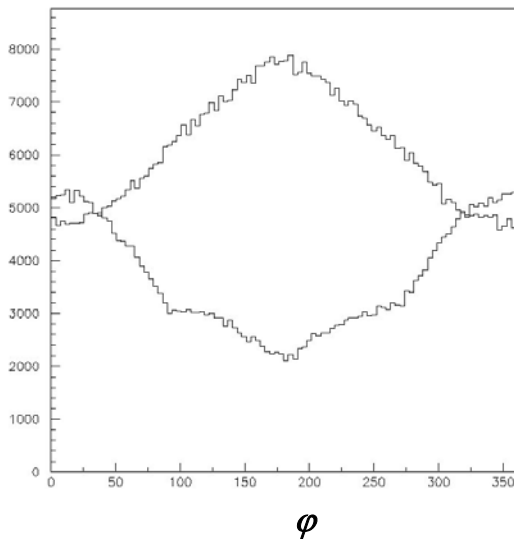


Figure 4.12a  $\varphi$  distributions of showers having  $\rho < 1$  (upper curve) and  $\rho > 1$  (lower curve) respectively.

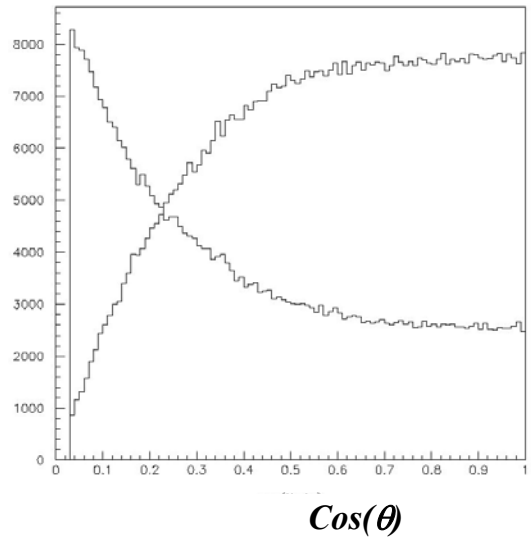


Figure 4.12b  $\theta$  distributions of showers having  $\rho < 1$  (upper curve) and  $\rho > 1$  (lower curve) respectively.

The distributions in atmosphere thickness ( $w_{field}$ ) and time span ( $t_{field}$ ) covered by the field of view are illustrated in Figure 4.13 showing the distribution of the showers in the  $(w_{field}, t_{field})$  plane for each of the two populations,  $\rho > 1$  and  $\rho < 1$ . The showers that are missed by the SDA are clearly seen to populate low values of  $w_{field}$ .

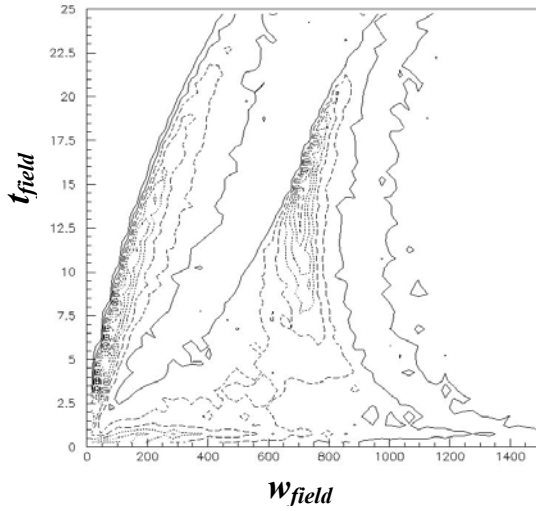


Figure 4.13a Distribution of the showers in the  $(w_{field}, t_{field})$  plane for showers having  $\rho > 1$

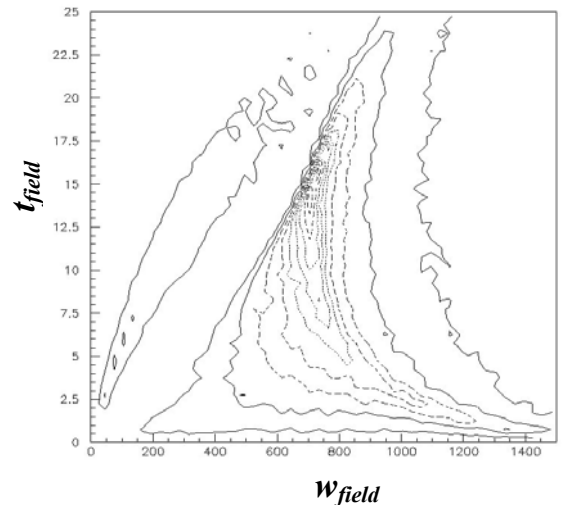


Figure 4.13b Distribution of the showers in the  $(w_{field}, t_{field})$  plane for showers having  $\rho < 1$

An accurate energy measurement implies a good coverage on either side of the shower maximum. Figure 4.14 shows the  $\rho$  distribution of showers having a window of at least  $\pm 300 \text{ g/cm}^2$  on either side of the shower maximum within the field of view (third curve from top). 24% of all hybrids fall in this category. An accurate measurement of  $a_{first}$  requires instead a good coverage of the early part of the shower development.

The  $\rho$  distribution of showers having a window of at least  $300 \text{ g/cm}^2$  in the first  $600 \text{ g/cm}^2$  of the shower development is also displayed on Figure 4.14 (second curve from top). 45% of all hybrids fall in this category. Showers satisfying both conditions are also shown (fourth curve from top). They amount to 11% of all hybrids.

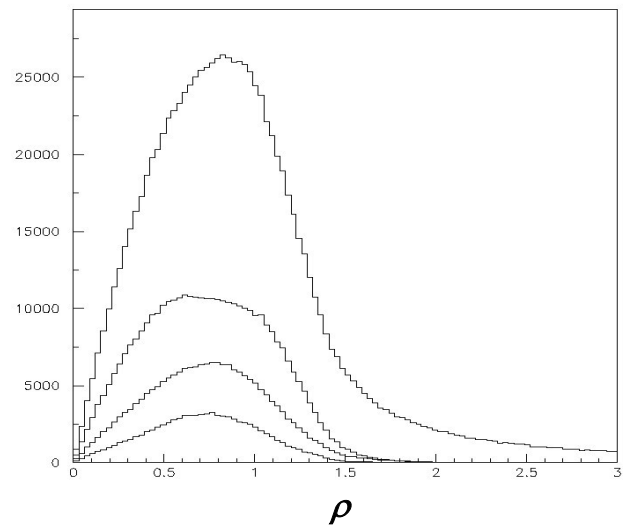


Figure 4.14  $\rho$  distributions of various shower samples. From top to bottom: all showers, good  $a_{first}$  measurement, good energy measurement, both (see text).

## SUMMARY AND PERSPECTIVES

The present work has concentrated on discussions of the methods used in the Pierre Auger Observatory to reconstruct showers induced by Ultra High Energy Cosmic Rays with particular emphasis on the sources of systematic errors that may affect the energy measurement. The systematic use of very simple simulation models and techniques has allowed for a clear understanding of the most relevant effects and for an easy identification of the parameters that define the quality of the energy measurement. Such a study needs to be completed by more detailed calculations using the very sophisticated simulation programs available to the Auger Collaboration. However such detailed studies are far less transparent and less convenient to interpret. Moreover the present study serves as a useful guide to an efficient use of the more sophisticated programs.

More time and effort were devoted to the study of the measurements performed in the fluorescence detector than in the surface detector. The reason is that the author is more closely associated with the former and will do his PhD work in collaboration with the Catania (Sicily) group of the PAO Collaboration, a group specialized in FD studies. In the SD case, the difficulty resides mostly in the indirect nature of the energy measurement that needs to be deduced from the transverse distribution of the density of secondaries on ground. A general and subtle problem is to understand how shower fluctuations may affect the energy measurement in such a case. While affecting also the longitudinal profile, they affect very little its integral and the calorimetric measurement offered by the FD is not expected to suffer of shower fluctuations as is the SD measurement. One of the tasks presently undertaken by the author is consequently a study of the effect of shower fluctuations on the longitudinal and transverse shower profiles, with particular emphasis on the implications concerning the SD energy measurement and the systematic errors that affect it.

Among the main results of the present study we may mention the identification of parameters that are relevant to the assessment of the quality of the measurements performed, such as the spans in time and in atmosphere thickness covered by the field of view of the FD eyes.

The difficulty of locating accurately the shower in the shower-detector plane in the case of monocular observation has been discussed in some detail, pointing to the importance of making binocular, or better hybrid observations. The compatibility between fluorescence and surface detections was briefly commented upon and found to be very good in the PAO design. Considerations on the necessity to extrapolate the observed shower profile outside the field of view have shed some light on the different requirements implied by a good energy measurement and a good evaluation of the nature of the primaries.

The problems arising when inferring the shower energy from the amount of light detected are major and have been discussed in detail in the literature. They include, among others, the subtraction of direct and scattered Cherenkov light and the calibration of the energy scale. While difficult to solve, they are in principle easy to understand, this is why they were not considered in the present study. A major advantage of hybrid detection is that such problems are completely absent from SD detection. However, SD detection has its own difficulties, essentially alien to fluorescence detection.

## REFERENCES

- [1] M. Nagano and A.A. Watson, *Rev. Mod. Phys.* 72 (2000) 689.
- [2] K. Greisen, *Phys. Rev. Lett.* 16 (1966) 748,  
Z.T. Zatsepin and V.A. Kuzmin, *Zh. Eksp. Teor. Fiz. Pis'ma Red.* 4 (1966) 144.
- [3] The Auger Collaboration, *Nucl. Instr. Meth. A* 523, 50 (2004).
- [4] J.W. Cronin, *The Highest Energy Cosmic Rays*, talk presented at TAUP 2003, Seattle, USA.
- [5] E. Parizot et al., editors, *Physique et Astrophysique du rayonnement cosmique*, Goutelas 2003, and references therein.
- [6] M. Hillas et al., 11th ICRC (1969) EAS-35.
- [7] M. Hillas et al., 12th ICRC (1969) EAS-18.
- [8] G. Medina-Tanco, in the proceedings of ICRC 2005.
- [9] R. Coy, *Astroparticle Physics* 6 (1997) 263.
- [10] R.M. Baltrusaitis et al., *Nucl. Inst. Meth.* A240 (1985) 410,  
D. Bird et al., *Nucl. Inst. Meth.*, A349 (1994) 592,  
D. Bird et al., *Astrophys. J.* 424 (1994) 491,  
G. Davidson and R. O'Neil, *J. Chem. Phys.* 41 (1964) 3946.
- [11] P. Sokolsky, *Am. Inst. Phys. Conf. Proc.* 433 (1998) 65.
- [12] T. K. Gaisser, *AIP Conf. Proc.* 558 (2001) 27.
- [13] “The Pierre Auger Observatory Progress and First Results”, Pierre Auger Collaboration, *Proceedings – 29th ICRC, Pune, India (2005)*.
- [14] T.K. Gaisser, “Cosmic rays and Particle Physics”, Cambridge University Press (1990).